EE 435

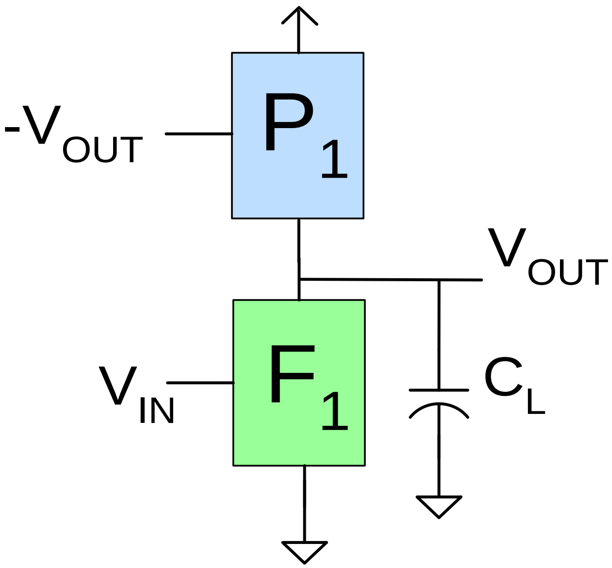
Lecture 21

Linearity of Bipolar and MOS Differential Pairs

Linearity of Common Source Amplifier

Offset Voltages

Gain Enhancement with Regenerative Feedback



$$A_{V0} = \frac{-g_{mF1}}{sC_L + g_{oF1} + g_{oP1} - g_{mP1}}$$

$$p = \frac{-g_{oF1} - g_{oP1} + g_{mP1}}{C_L}$$

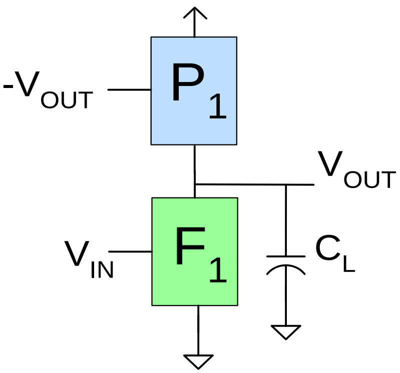
If $g_{mP1} > g_{oF1} + g_{oP1}$, the pole will be in the RHP !!

The feedback **performance can actually be enhanced** if the open-loop amplifier with gain reversal is unstable

Why?

Review from last lecture

Gain Enhancement with Regenerative Feedback

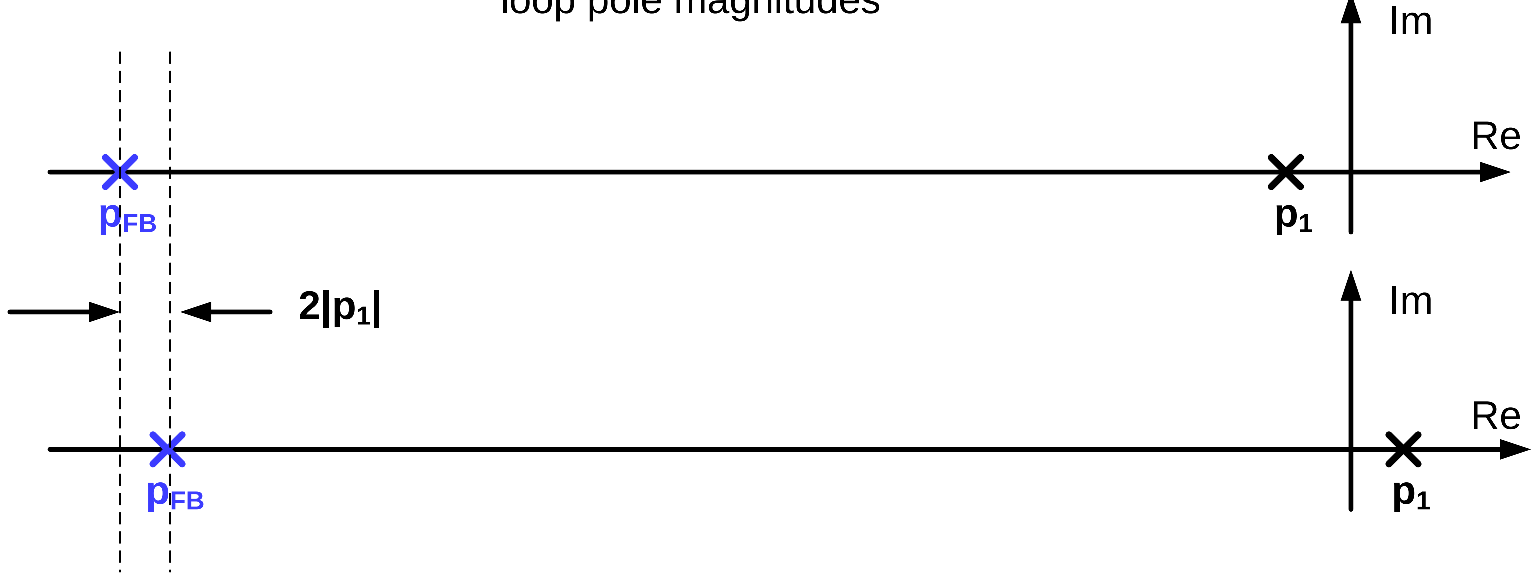


It will be shown that a feedback amplifier with dc gain reversing with pole is usually stable even if the open-loop Op amp is unstable!

How?

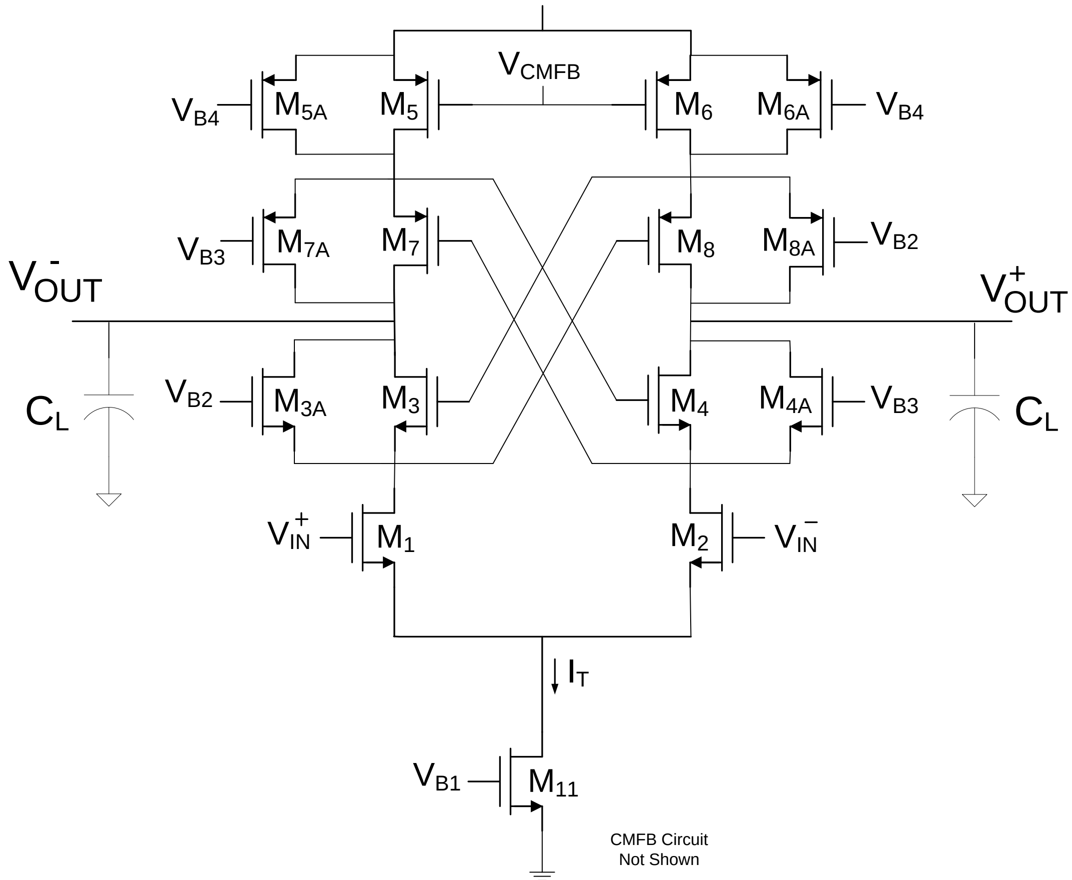
$$p_{FB} = \begin{cases} p_1\left(1+\beta A_{V0}\right) & \text{for } p_1 < 0 \\ p_1\left(1-\left|\beta A_{V0}\right|\right) & \text{for } p_1 > 0 \end{cases}$$

Open-Loop and Closed-Loop Pole Plot for equal open-loop pole magnitudes



Review from last lecture

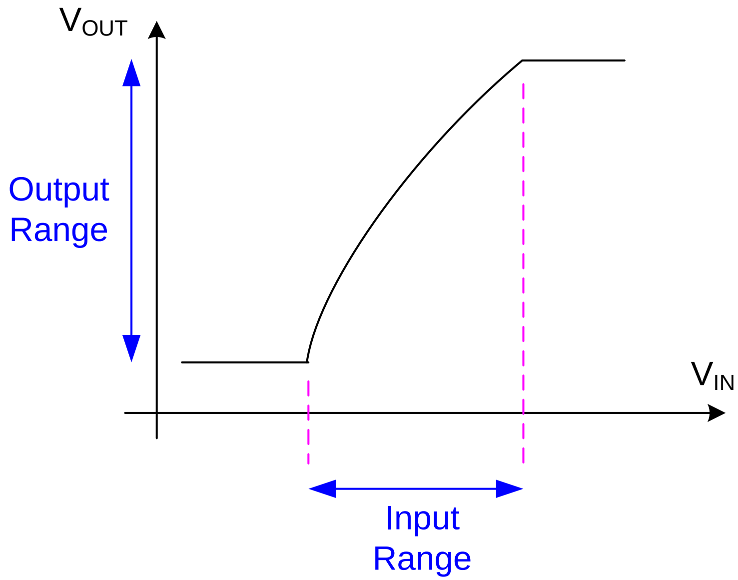
Another Positive Feedback Amplifier

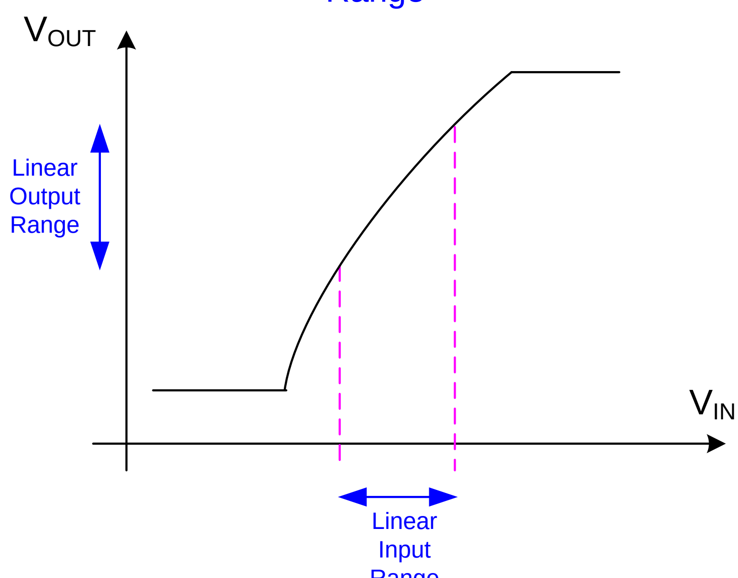


- Regenerative feedback can be to either quarter circuit or counterpart circuit
- Regenerative feedback to cascode devices can significantly reduce the magnitude of the negative conductance term

Review from last lecture

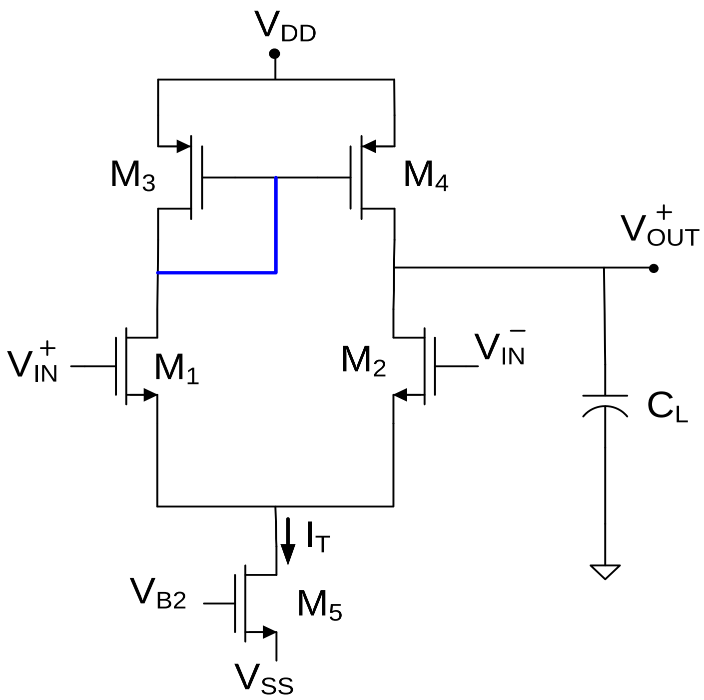
Signal Swing and Linearity



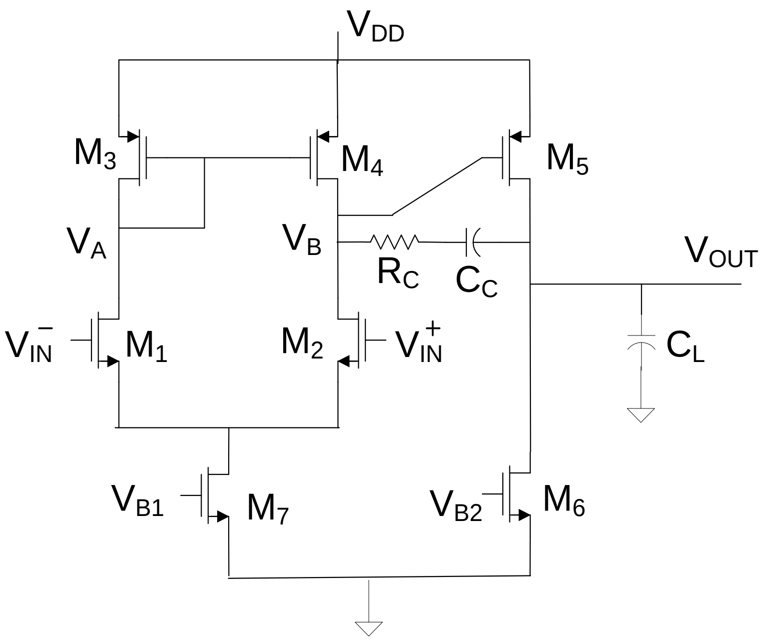


Review from last lecture

Linearity of Amplifiers



Single-Stage



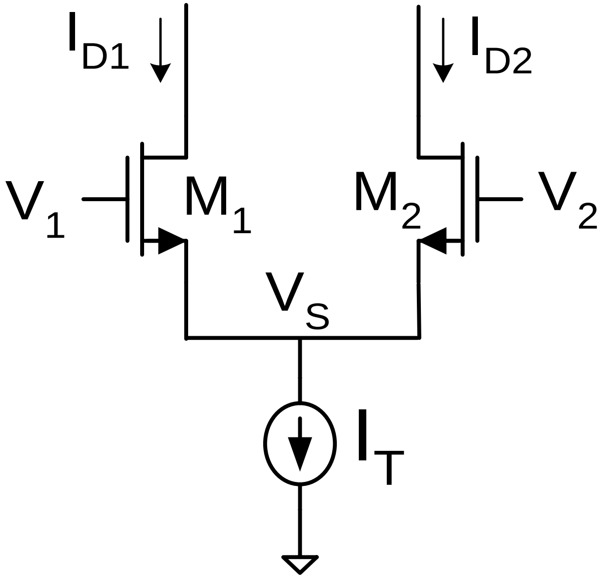
Two-Stage

Linearity of differential pair of major concern

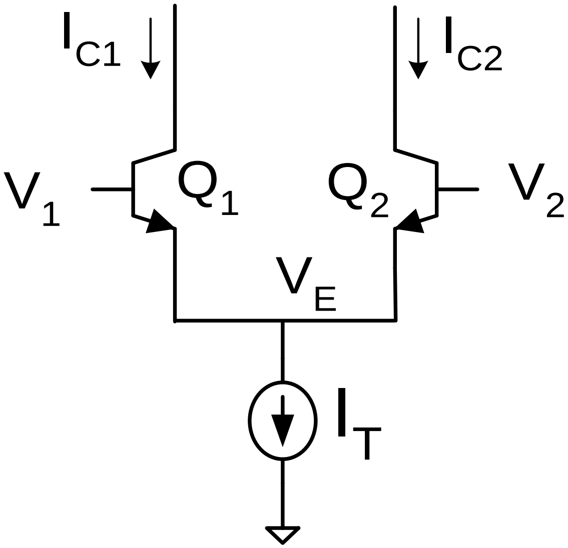
Linearity of common-source amplifier is of major concern (since signals so small at output of differential pair)

Review from last lecture

Differential Input Pairs



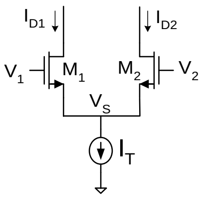
MOS Differential Pair



Bipolar Differential Pair

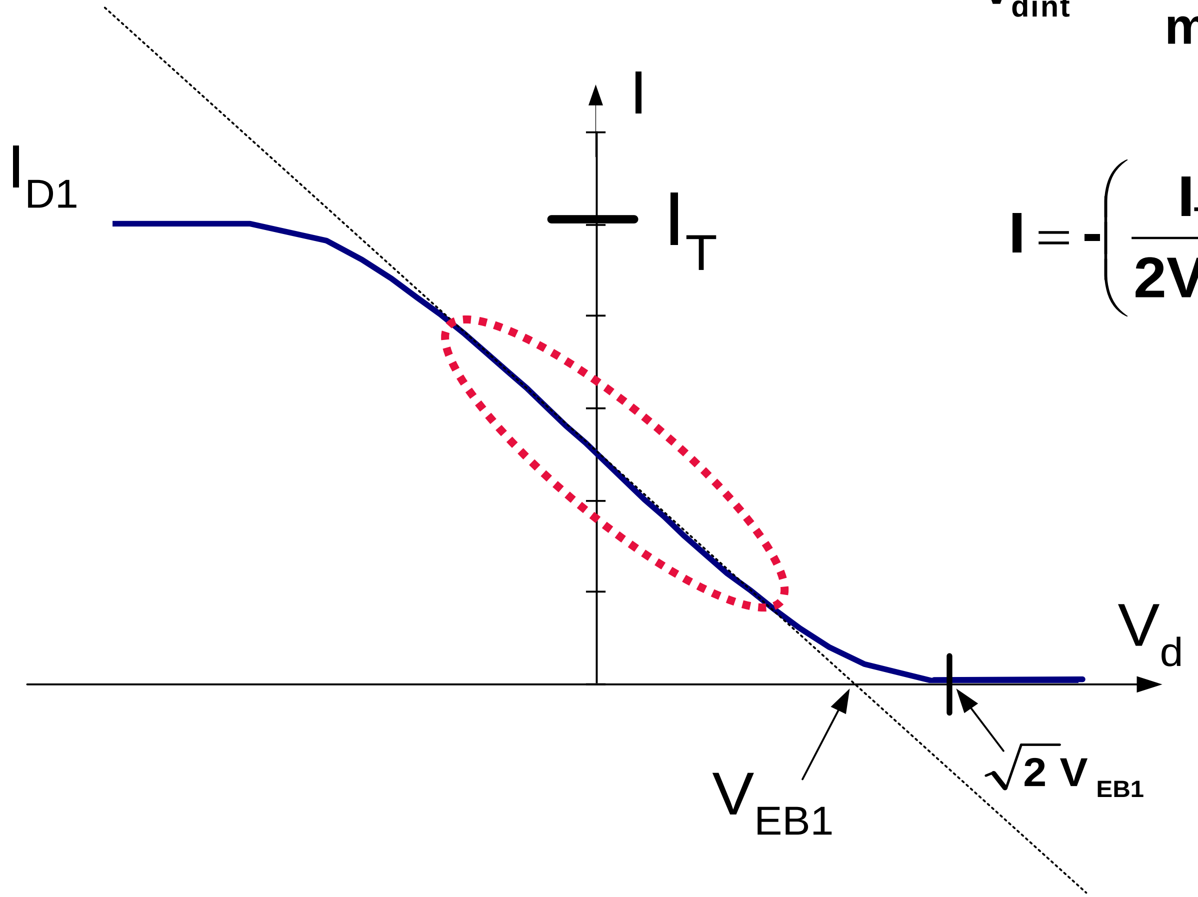
Review from last lecture

How linear is the amplifier ?

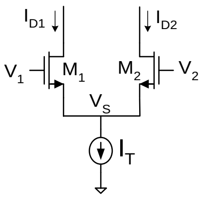


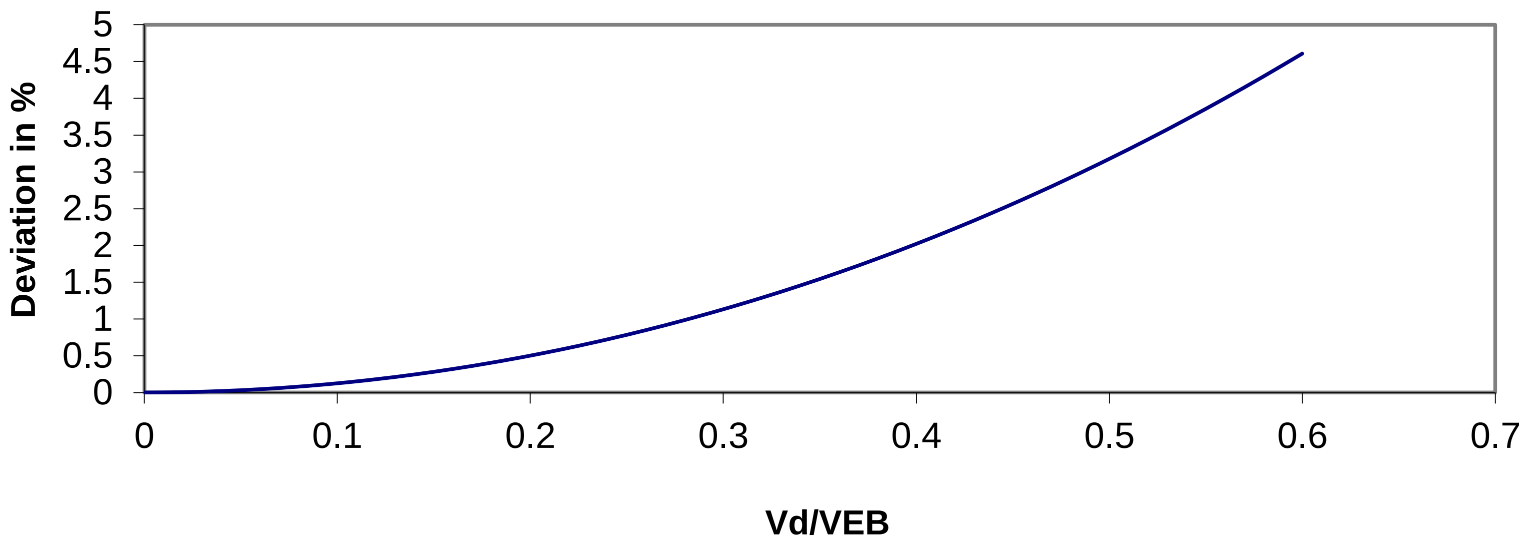
$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m} = V_{EB1}$$

$$I = -\left(\frac{I_T}{2V_{EB1}}\right)V_d + \frac{I_T}{2}$$



How linear is the amplifier ?





Vd/VEB	θ		Vd/VEB	θ		Vd/VEB	θ
0.02	0.005		0.22	0.607		0.42	2.23
0.04	0.020		0.24	0.723		0.44	2.45
0.06	0.045		0.26	0.849		0.46	2.68
0.08	0.080		0.28	0.985		0.48	2.92
0.1	0.125		0.3	1.13		0.5	3.18
0.12	0.180		0.32	1.29		0.52	3.44
0.14	0.245		0.34	1.46		0.54	3.71
0.16	0.321		0.36	1.63		0.56	4.00
0.18	0.406		0.38	1.82		0.58	4.30
0.2	0.501		0.4	2.02		0.6	4.61

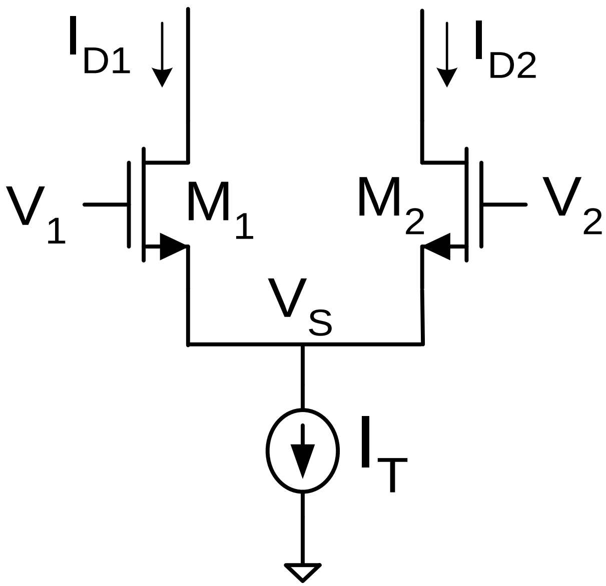
How linear is the amplifier ?

Distortion in the differential pair is another useful metric for characterizing linearity of I_{D1} and I_{D2} with sinusoidal differential excitation

Consider again the differential pair and assume excited differentially with

$$V_2 = \frac{V_d}{2} \qquad V_1 = -\frac{V_d}{2}$$

and assume $V_d = V_m \sin(\omega t)$



$$\mathbf{V_d = V_2 - V_1}$$

Recall:

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$

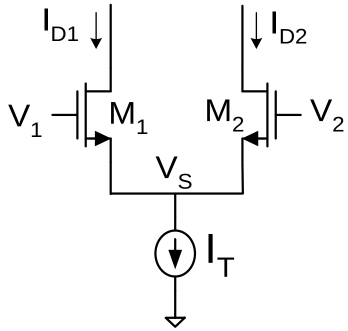
Define (strictly for notational convenience)

$$\theta = \frac{\mu C_{OX} W}{2L}$$

Thus can express as

$$\sqrt{\theta} V_d = \sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}$$

How linear is the amplifier ?



$$V_d = V_m \sin(\omega t) \qquad \theta = \frac{\mu C_{OX} W}{2L}$$

$$\sqrt{\theta} V_d = \sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}$$

Squaring, regrouping, and squaring we obtain

$$\theta V_d^2 = I_{D2} + (I_T - I_{D2}) - 2\sqrt{I_{D2}}\sqrt{I_T - I_{D2}}$$

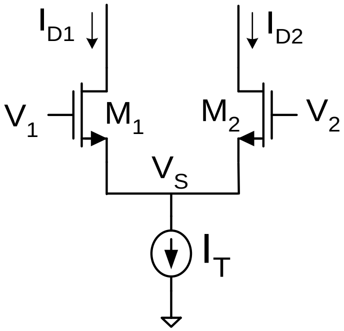
$$\theta V_d^2 = I_T - 2\sqrt{I_{D2}}\sqrt{I_T - I_{D2}}$$

$$\left(\theta V_d^2 - I_T\right)^2 = 4 I_{D2}\left(I_T - I_{D2}\right)$$

This latter equation can be expressed as a second-order polynomial in I_{D2} as

$$I_{D2}^2 - I_{D2} I_T + \left(\frac{\theta V_d^2 - I_T}{2}\right)^2 = 0$$

How linear is the amplifier ?



and assume $V_d = V_m \sin(\omega t)$ $\theta = \frac{\mu C_{OX} W}{2L}$

$$I_{D2}^2 - I_{D2} I_T + \left(\frac{\theta V_d^2 - I_T}{2} \right)^2 = 0$$

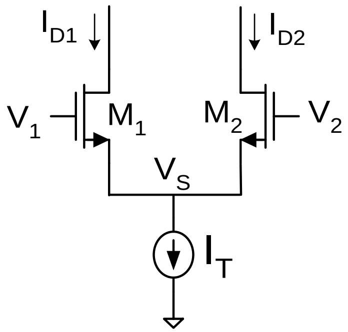
Solving, we obtain

$$I_{D2} = \frac{I_T}{2} + \sqrt{\left(\frac{I_T}{2} \right)^2 - \left(\frac{\theta V_d^2 - I_T}{2} \right)^2}$$

$$I_{D2} = \frac{I_T}{2} + \sqrt{\left(\frac{I_T}{2} \right)^2 - \left(\frac{\theta V_d^2}{2} \right)^2 - \left(\frac{I_T}{2} \right)^2 + \frac{\theta I_T}{2} V_d^2}$$

$$I_{D2} = \frac{I_T}{2} + \sqrt{\frac{\theta I_T}{2} V_d^2 - \left(\frac{\theta V_d^2}{2} \right)^2}$$

How linear is the amplifier ?



and assume $V_d=V_m\sin(\omega t)$ $\qquad \theta = \dfrac{\mu C_{OX} W}{2L}$

$$I_{D2} = \frac{I_T}{2} + \sqrt{\frac{\theta I_T}{2} V_d^2 - \left(\frac{\theta V_d^2}{2}\right)^2}$$

This can be expressed as

$$I_{D2} = \frac{I_T}{2} + V_d \sqrt{\frac{\theta I_T}{2}} \sqrt{1 - V_d^2 \frac{\theta}{2 I_T}}$$

Recall for x small

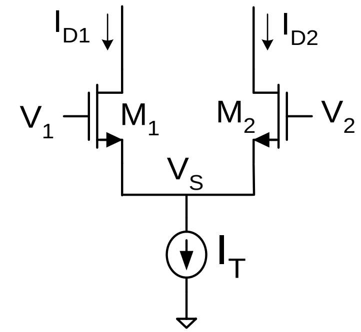
$$\sqrt{1-x} \cong 1 - \frac{x}{2} - \frac{x^2}{8} + \ldots.$$

Using a Truncated Taylor's series, we obtain:

$$I_{D2} \simeq \frac{I_T}{2} + V_d \sqrt{\frac{\theta I_T}{2}} \left(1 - V_d^2 \frac{\theta}{4 I_T}\right)$$

Note this has no second-order term thus the dominant distortion when $V_d=V_m\sin(\omega t)$ will be due to the third-order term

How linear is the amplifier ?



$$I_{D2} \simeq \frac{I_T}{2} + V_d \sqrt{\frac{\theta I_T}{2}} \left(1 - V_d^2 \frac{\theta}{4 I_T}\right)$$

$$\theta = \frac{\mu C_{OX} W}{2L}$$

Substituting in $V_d = V_m \sin(\omega t)$

$$I_{D2} \simeq \frac{I_T}{2} + V_m \sin(\omega t) \sqrt{\frac{\theta I_T}{2}} \left(1 - V_m^2 \sin^2(\omega t) \frac{\theta}{4 I_T}\right)$$

$$I_{D2} \simeq \frac{I_T}{2} + \left[V_m \sqrt{\frac{\theta I_T}{2}}\right] \sin(\omega t) - \left[V_m^3 \frac{\theta^{\frac{3}{2}}}{4\sqrt{2}\sqrt{I_T}}\right] \sin^3(\omega t)$$

$$\sin^3(\omega t) = \frac{3}{4}\sin(\omega t) - \frac{1}{4}\sin(3\omega t)$$

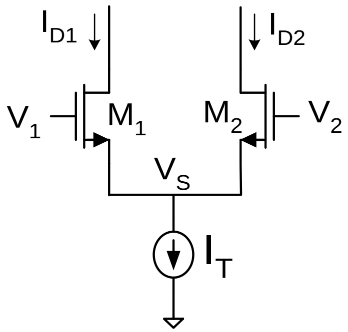
$$I_{D2} \simeq \frac{I_T}{2} + \left[V_m \sqrt{\frac{\theta I_T}{2}}\right] \sin(\omega t) - \left[V_m^3 \frac{\theta^{\frac{3}{2}}}{4\sqrt{2}\sqrt{I_T}}\right] \left[\frac{3}{4}\sin(\omega t) - \frac{1}{4}\sin(3\omega t)\right]$$

$$I_{D2} \simeq \frac{I_T}{2} + \left[V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}}\right] \sin(\omega t) + \left[V_m^3 \frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}}\right] \left[\sin(3\omega t)\right]$$

How linear is the amplifier ?

$$I_{D2} \simeq \frac{I_T}{2} + V_d\sqrt{\frac{\theta I_T}{2}}\left(1 - V_d^2\frac{\theta}{4I_T}\right)$$

$$\theta = \frac{\mu C_{OX} W}{2L}$$



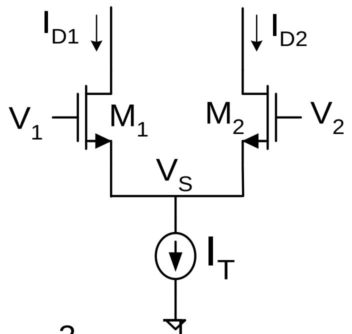
$$I_{D2} \simeq \frac{I_T}{2} + \left[V_m\sqrt{\frac{\theta I_T}{2}} - V_m^3\frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}}\right]\sin(\omega t) + \left[V_m^3\frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}}\right]\left[\sin(3\omega t)\right]$$

Note this has no second-order harmonic term thus the dominant distortion when $V_d = V_m \sin(\omega t)$ will be due to the third-order harmonic

$$I_{D2} \simeq a_0 + a_1\sin(\omega t) + a_3(3\omega t)$$

$$a_1 = \left[V_m\sqrt{\frac{\theta I_T}{2}} - V_m^3\frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}}\right] \qquad a_3 = \left[\frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}}\right]V_m^3$$

How linear is the amplifier ?



$$I_{D2} \simeq a_0 + a_1 \sin(\omega t) + a_3 \sin(3\omega t)$$

$$a_1 = \left[V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] \qquad a_3 = \left[\frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] V_m^3$$

$$THD = 20\log\left(\frac{\sqrt{\sum_{k=2}^{\infty} a_k^2}}{a_1} \right)$$

For low distortion want THD a large negative number

Substituting in we obtain

$$THD = 20\log\left(\frac{\frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} V_m^3}{V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}}} \right)$$

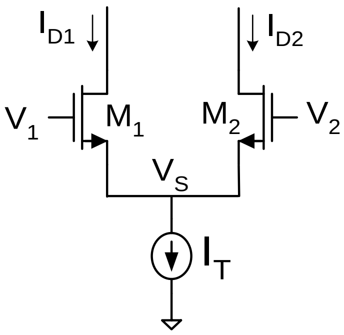
where $\theta = \frac{\mu C_{OX} W}{2L}$

This expression gives little insight.

Consider expression in the practical parameter domain:

$$I_T = \frac{\mu C_{OX} W}{L} V_{EB1}^2$$

How linear is the amplifier ?



$$I_{D2} \simeq a_0 + a_1 \sin(\omega t) + a_3 \sin(3\omega t)$$

$$THD = 20\log\left(\frac{\dfrac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} V_m^3}{V_m\sqrt{\dfrac{\theta I_T}{2}} - V_m^3 \dfrac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}}}\right)$$

$$\theta = \frac{\mu C_{OX} W}{2L}$$

$$I_T = \frac{\mu C_{OX} W}{L} V_{EB1}^2$$

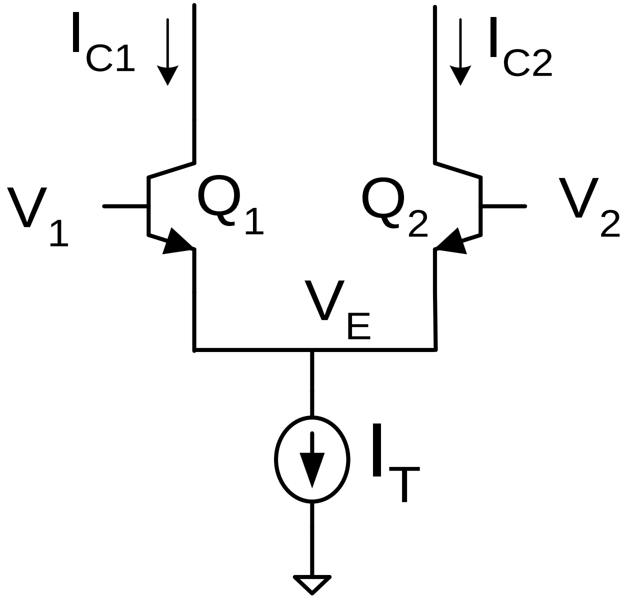
Eliminating I_T and θ, we obtain

$$THD = -20\log\left(32\left(\frac{V_{EB1}}{V_m}\right)^2 - 3\right)$$

V_m / V_{EB1}	THD (dB)
2.5	-6.52672
1	-29.248
0.5	-41.9382
0.25	-54.1344
0.1	-70.0949
0.05	-82.1422
0.025	-94.1849
0.01	-110.103

Thus to minimize THD, want V_{EB} large and V_m small

Bipolar Differential Pair



$$\left.\begin{aligned} I_{C1} &= J_S A_{E1} e^{\frac{V_1 - V_E}{V_t}} \\ I_{C2} &= J_S A_{E2} e^{\frac{V_2 - V_E}{V_t}} \\ I_{C1} + I_{C2} &= I_T \end{aligned}\right\}$$

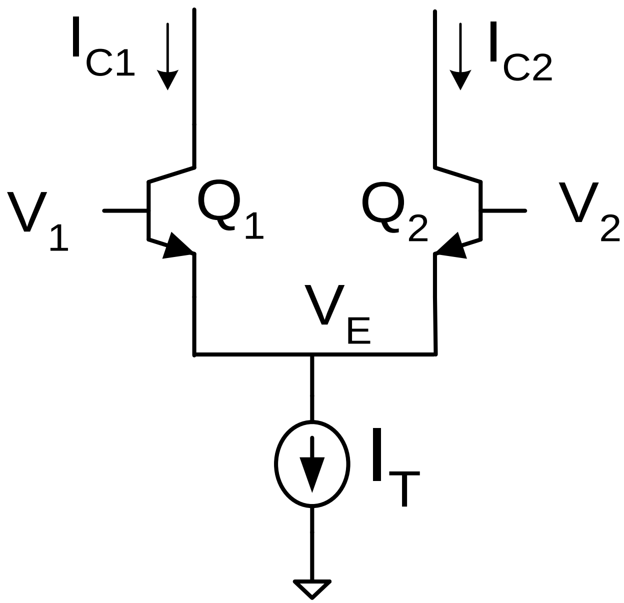
$$V_1 = V_E + V_t \ln\left(\frac{I_{C1}}{J_S A_{E1}}\right)$$

$$V_2 = V_E + V_t \ln\left(\frac{I_{C2}}{J_S A_{E2}}\right)$$

$$V_d = V_2 - V_1$$

$$V_d = V_t\left(\ln\left(\frac{I_{C2}}{J_S A_{E2}}\right) - \ln\left(\frac{I_{C1}}{J_S A_{E1}}\right)\right) \overset{A_{E1}=A_{E2}}{=} V_t \ln\left(\frac{I_{C2}}{I_{C1}}\right)$$

Bipolar Differential Pair



$$V_d = V_2 - V_1$$

$$V_d = V_t\left(\ln\left(\frac{I_{C2}}{J_S A_{E2}}\right) - \ln\left(\frac{I_{C1}}{J_S A_{E1}}\right)\right) \overset{A_{E1}=A_{E2}}{=} V_t \ln\left(\frac{I_{C2}}{I_{C1}}\right)$$

$$V_d = V_t \ln\left(\frac{I_T - I_{C1}}{I_{C1}}\right)$$

$$V_d = V_t \ln\left(\frac{I_{C2}}{I_T - I_{C2}}\right)$$

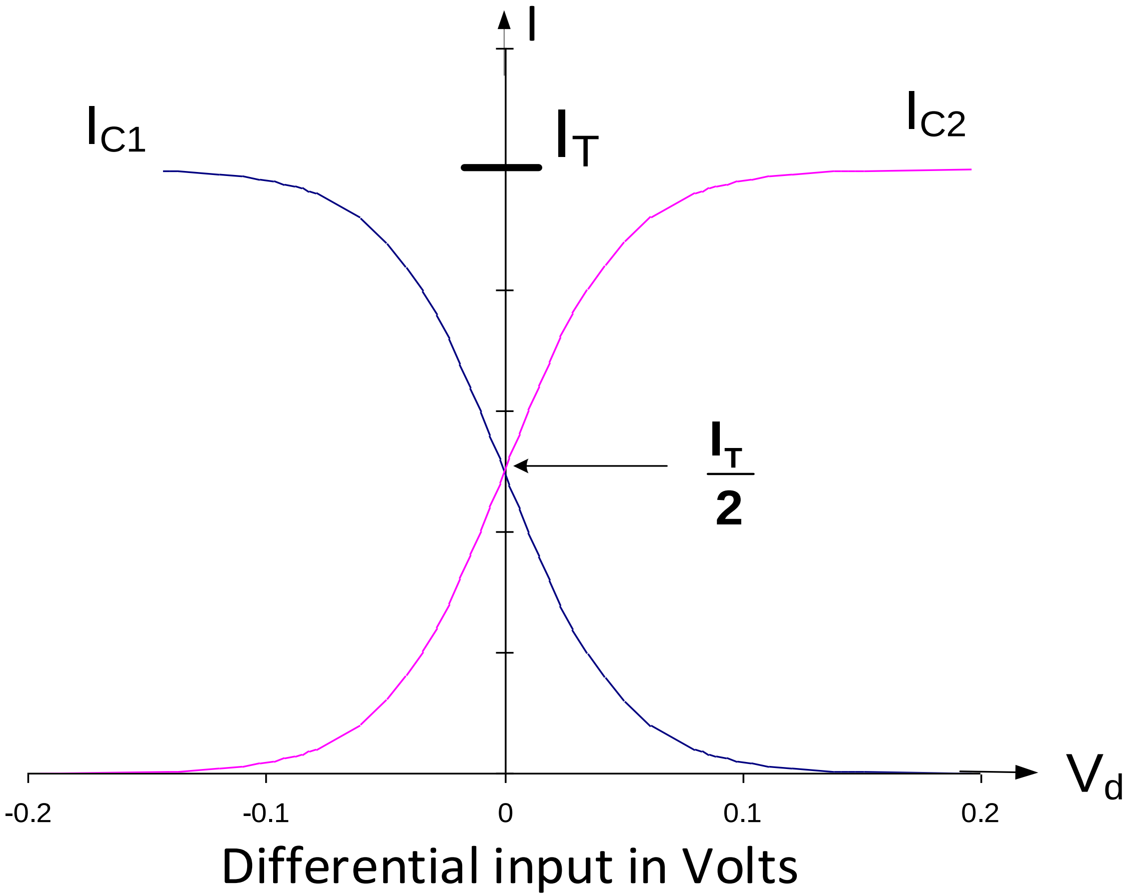
At $I_{C1}=I_{C2}=I_T/2$, $V_d=0$

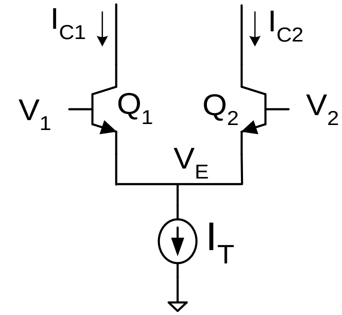
As I_{C1} approaches 0, V_d approaches infinity

As I_{C1} approaches I_T, V_d approaches minus infinity

Transition much steeper than for MOS case

Transfer Characteristics of Bipolar Differential Pair



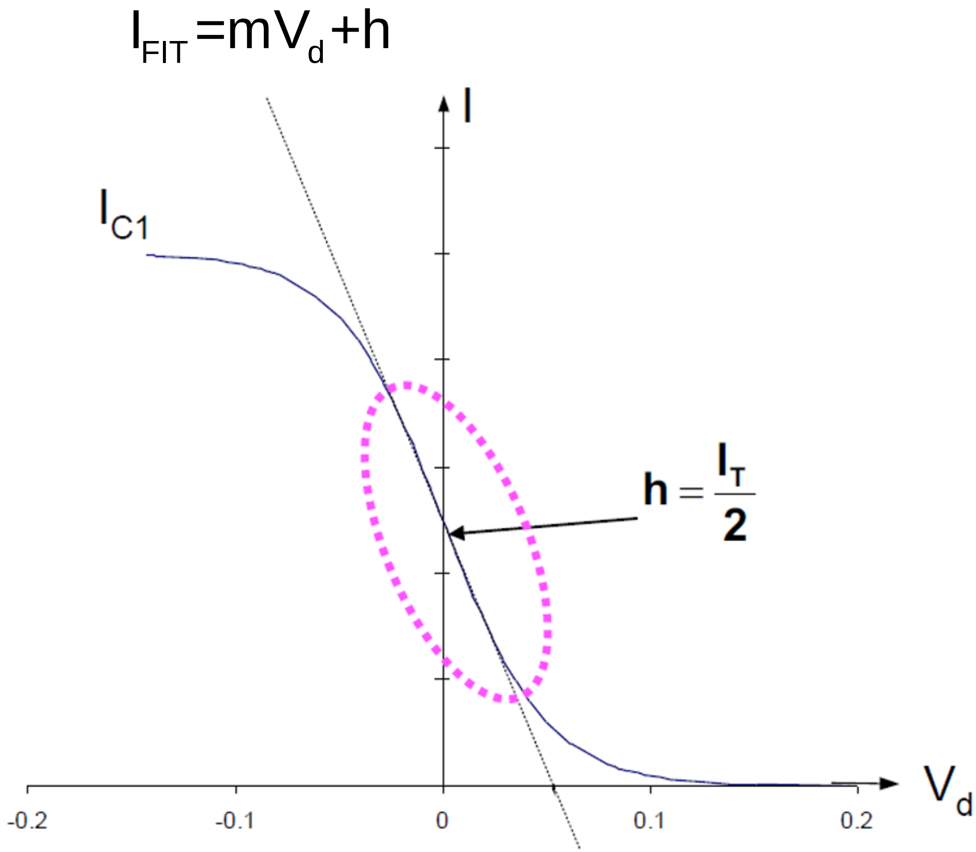


$$\mathbf{V_d = V_t \ln\left(\frac{I_T - I_{C1}}{I_{C1}}\right)}$$

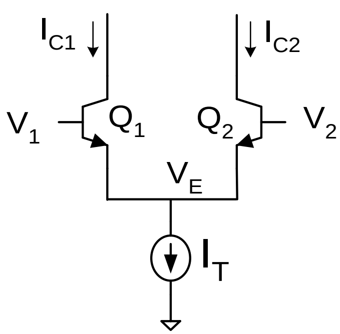
Transition much steeper than for MOS case

Asymptotic Convergence to 0 and I_T

Signal Swing and Linearity of Bipolar Differential Pair



$$V_{d\,\text{int}} = -\frac{h}{m} = ?$$



$$\mathbf{V_d = V_t \ln\left(\frac{I_T - I_{C1}}{I_{C1}}\right)}$$

$$m = \left.\frac{\partial I_{C1}}{\partial V_d}\right|_{Q-\text{point}}$$

Q-pt = (0,h)

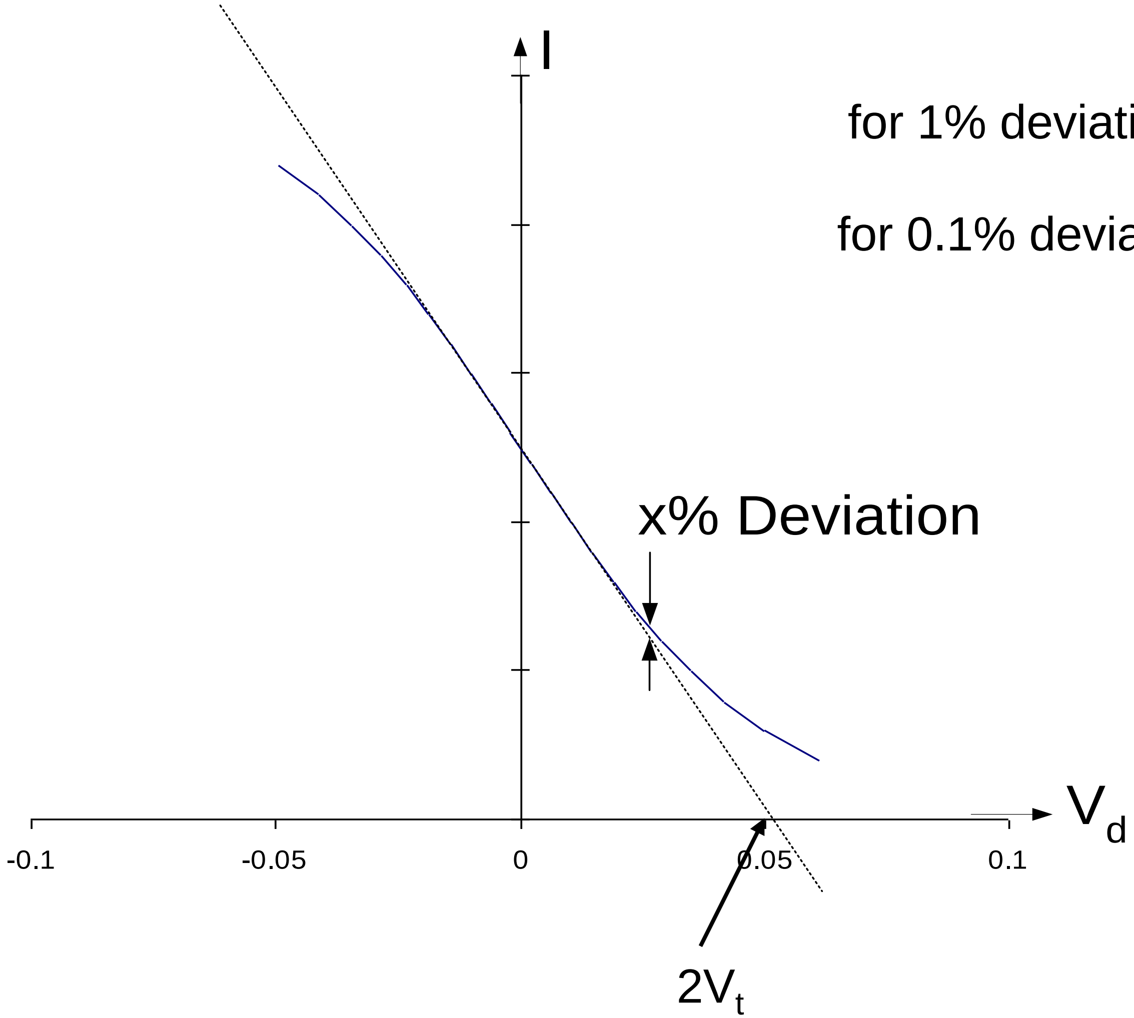
$$\left.\frac{\partial V_d}{\partial I_{C1}}\right|_{Q=\text{point}} = -V_t \left.\frac{I_T}{I_{C1}\left(I_T - I_{C1}\right)}\right|_{I_{C1}=\frac{I_T}{2}}$$

$$\left.\frac{\partial V_d}{\partial I_{C1}}\right|_{Q=\text{point}} = -\frac{4V_t}{I_T}$$

$$I_{FIT} = -\frac{I_T}{4V_t}V_d + \frac{I_T}{2}$$

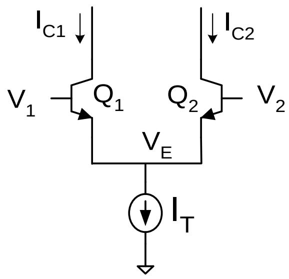
$$V_{d\,\text{int}} = -\frac{h}{m} = 2V_t$$

Signal Swing and Linearity of Bipolar Differential Pair

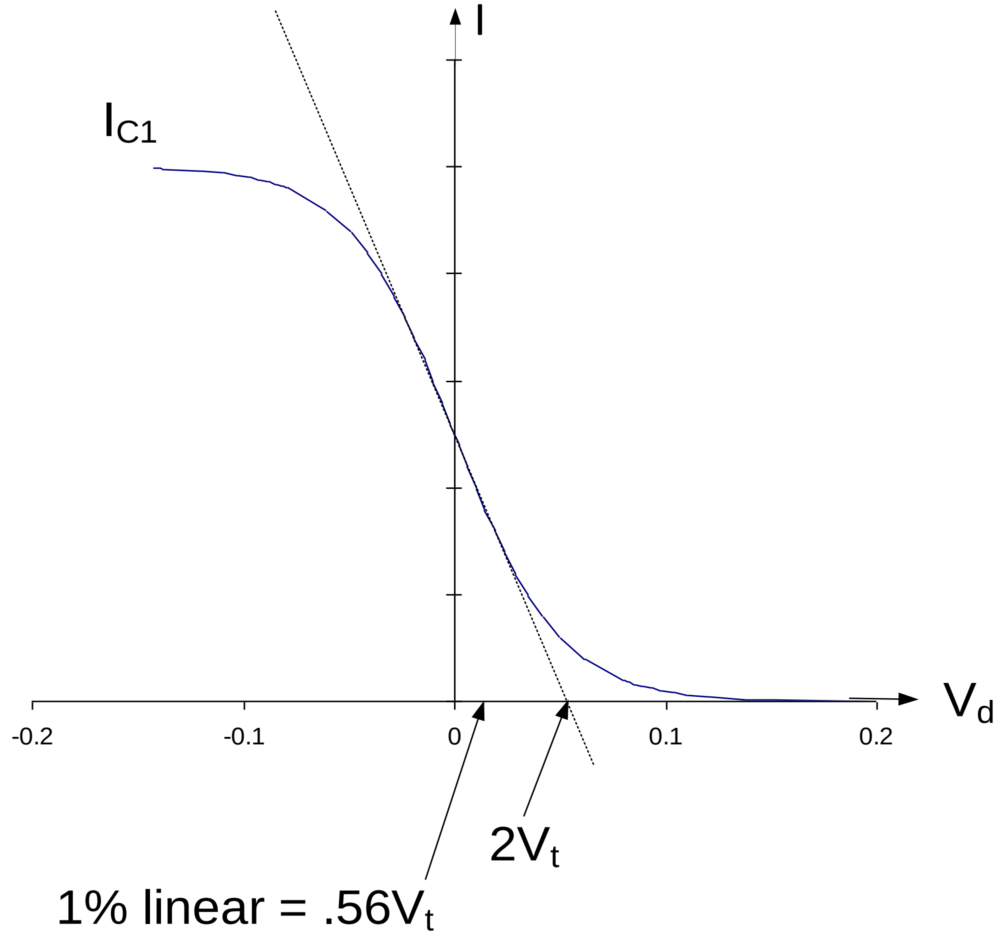


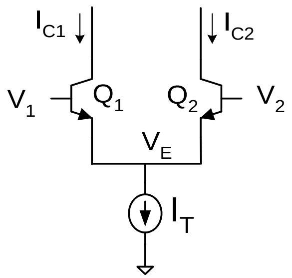
for 1% deviation, V_d=.56V_t

for 0.1% deviation, V_d=.27V_t



Signal Swing and Linearity of Bipolar Differential Pair





Note V_d axis intercept for BJT pair typically much smaller than for MOS pair (V_{EB}) but designer has no control of intercept for BJT pair

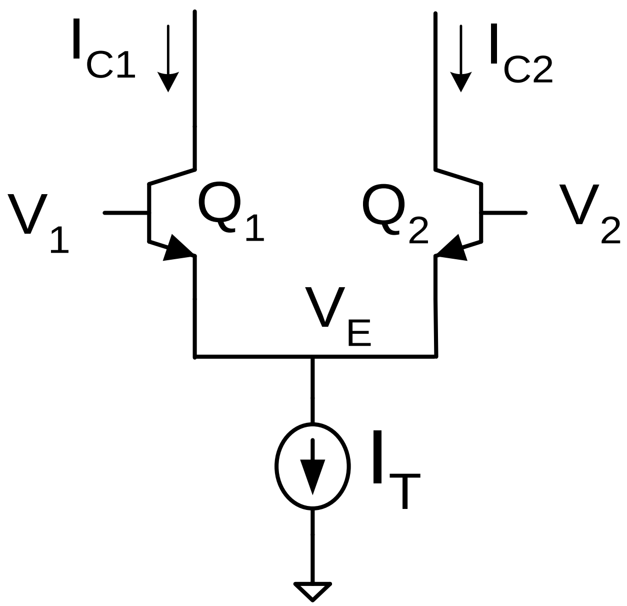
How linear is the amplifier ?

Distortion in the differential pair is another useful metric for characterizing linearity of I_{C1} and I_{c2} with sinusoidal differential excitation

Consider again the differential pair and assume excited differentially with

$$V_2 = \frac{V_d}{2} \qquad V_1 = -\frac{V_d}{2}$$

and assume $V_d = V_m \sin(\omega t)$



$$\mathbf{V_d = V_2 - V_1}$$

Recall:

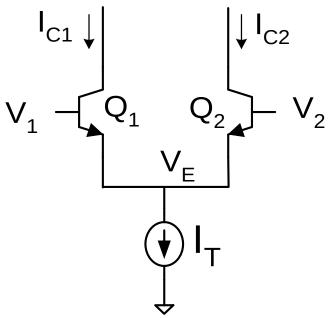
$$V_d = V_t \ln\left(\frac{I_T - I_{C1}}{I_{C1}}\right)$$

Thus can express as

$$e^{\frac{V_d}{V_t}} = \frac{I_T - I_{C1}}{I_{C1}}$$

$$I_{C1} = I_T\left(1 + e^{\frac{V_d}{V_t}}\right)^{-1}$$

How linear is the amplifier ?



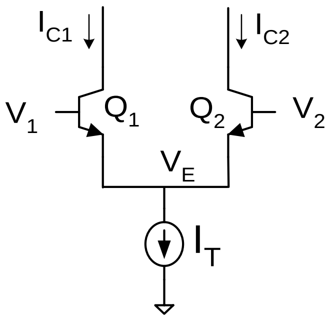
$$I_{C1} = I_T \left(1 + e^{\frac{V_d}{V_t}} \right)^{-1}$$

$$V_d = V_m \sin(\omega t)$$

Consider a Taylor's Series Expansion

$$I_{C1} = I_{C1}\Big|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} V_d + \frac{1}{2!}\frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} V_d^2 + \frac{1}{3!}\frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} V_d^3 \quad + H.O.T$$

How linear is the amplifier ?



$$I_{C1} = I_T \left(1 + e^{\frac{V_d}{V_t}}\right)^{-1} \qquad V_d = V_m \sin(\omega t)$$

$$I_{C1} = I_{C1}\Big|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} V_d + \frac{1}{2!}\frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} V_d^2 + \frac{1}{3!}\frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} V_d^3 + H.O.T$$

$$\frac{\partial I_{C1}}{\partial V_d} = -\frac{I_T}{V_t}\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}}$$

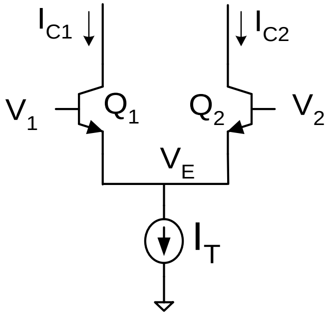
$$\frac{\partial^2 I_{C1}}{\partial V_d^2} = -\frac{I_T}{V_t}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} \frac{1}{V_t} - 2\, e^{\frac{V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3} e^{\frac{V_d}{V_t}} \frac{1}{V_t}\right]$$

$$\frac{\partial^2 I_{C1}}{\partial V_d^2} = -\frac{I_T}{V_t^2}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} - 2\, e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3}\right]$$

$$\frac{\partial^3 I_{C1}}{\partial V_d^3} = -\frac{I_T}{V_t^2}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} \frac{1}{V_t} - 2e^{\frac{V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3} e^{\frac{V_d}{V_t}} \frac{1}{V_t} + 6\, e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-4} e^{\frac{V_d}{V_t}} \frac{1}{V_t} - 2e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3} \frac{2}{V_t}\right]$$

$$\frac{\partial^3 I_{C1}}{\partial V_d^3} = -\frac{I_T}{V_t^3}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} - 2e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3} + 6e^{\frac{3V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-4} - 4e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3}\right]$$

How linear is the amplifier ?



$V_d = V_m \sin(\omega t)$

$$I_{C1} = I_{C1}\Big|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} V_d + \frac{1}{2!}\frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} V_d^2 + \frac{1}{3!}\frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} V_d^3 + H.O.T$$

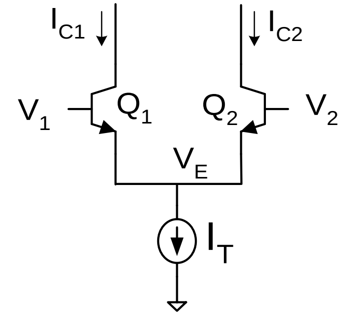
$$\frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} = -\frac{I_T}{V_t}\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}}\Big|_{V_d==0} = -\frac{I_T}{V_t}(2)^{-2} = -\frac{I_T}{4V_t}$$

$$\frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} = -\frac{I_T}{V_t^2}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} - 2\,e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3}\right]\Big|_{V_d=0} = -\frac{I_T}{V_t^2}\left[(2)^{-2} - 2(2)^{-3}\right] = 0$$

$$\frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} = -\frac{I_T}{V_t^3}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} - 2e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3} + 6e^{\frac{3V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-4} - 4e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3}\right]\Big|_{V_d=0} = -\frac{I_T}{V_t^3}\left[(2)^{-2} - 2(2)^{-3} + 6(2)^{-4} - 4(2)^{-3}\right] = \frac{I_T}{8V_t^3}$$

$$\frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} = -\frac{I_T}{4V_t} \qquad \frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} = 0 \qquad \frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} = \frac{I_T}{8V_t^3}$$

How linear is the amplifier ?



$V_d = V_m \sin(\omega t)$

$$I_{C1} = I_{C1}\Big|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} V_d + \frac{1}{2!}\frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} V_d^2 + \frac{1}{3!}\frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} V_d^3 + H.O.T$$

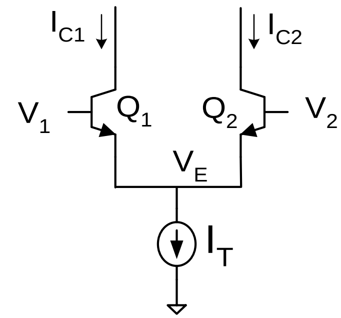
$$\frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} = -\frac{I_T}{4V_t} \qquad \frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} = 0 \qquad \frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} = \frac{I_T}{8V_t^3}$$

$$I_{C1} \cong \frac{I_T}{2} - \frac{I_T}{4V_t} V_d + \frac{I_T}{48V_t^3} V_d^3$$

$$I_{C1} \cong \frac{I_T}{2} - \frac{I_T}{4V_t} V_m \sin(\omega t) + \frac{I_T}{48V_t^3} V_m^3 \sin^3(\omega t)$$

$$\sin^3(\omega t) = \frac{3}{4}\sin(\omega t) - \frac{1}{4}\sin(3\omega t)$$

How linear is the amplifier ?



$$V_d = V_m \sin(\omega t)$$

$$I_{C1} = I_{C1}\Big|_{V_d=0} + \left.\frac{\partial I_{C1}}{\partial V_d}\right|_{V_d=0} V_d + \frac{1}{2!}\left.\frac{\partial^2 I_{C1}}{\partial V_d^2}\right|_{V_d=0} V_d^2 + \frac{1}{3!}\left.\frac{\partial^3 I_{C1}}{\partial V_d^3}\right|_{V_d=0} V_d^3 + H.O.T$$

$$I_{C1} \cong \frac{I_T}{2} - \frac{I_T}{4V_t} V_m \sin(\omega t) + \frac{I_T}{48V_t^3} V_m^3 \left[\frac{3}{4}\sin(\omega t) - \frac{1}{4}\sin(3\omega t)\right]$$

$$I_{C1} \cong \frac{I_T}{2} + \left[\frac{3I_T}{4 \bullet 48V_t^3} V_m^3 - \frac{I_T}{4V_t} V_m\right]\sin(\omega t) - \frac{I_T}{4 \bullet 48V_t^3} V_m^3 \sin(3\omega t)$$

Thus:

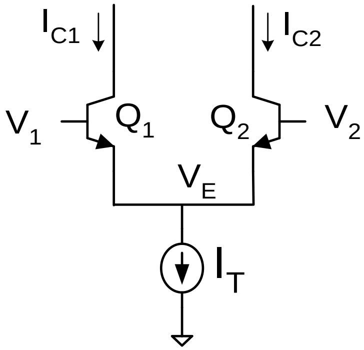
$$THD = 20\log\left(\frac{V_m^2}{\left[48V_t^2 - 3V_m^2\right]}\right)$$

or, equivalently

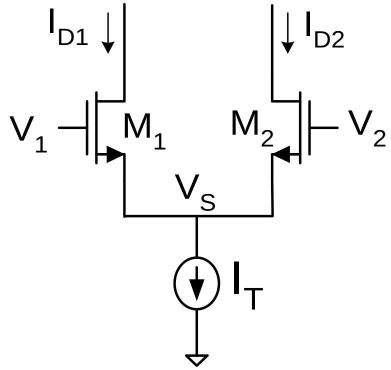
$$THD = -20\log\left(48\left(\frac{V_t}{V_m}\right)^2 - 3\right)$$

V_m/V_t	THD (dB)
2.5	-13.4049
1	-33.0643
0.5	-45.5292
0.25	-57.6732
0.1	-73.6194
0.05	-85.6647
0.025	-97.7069
0.01	-113.625

Comparison of Distortion in BJT and MOSFET Pairs



$V_d = V_m \sin(\omega t)$



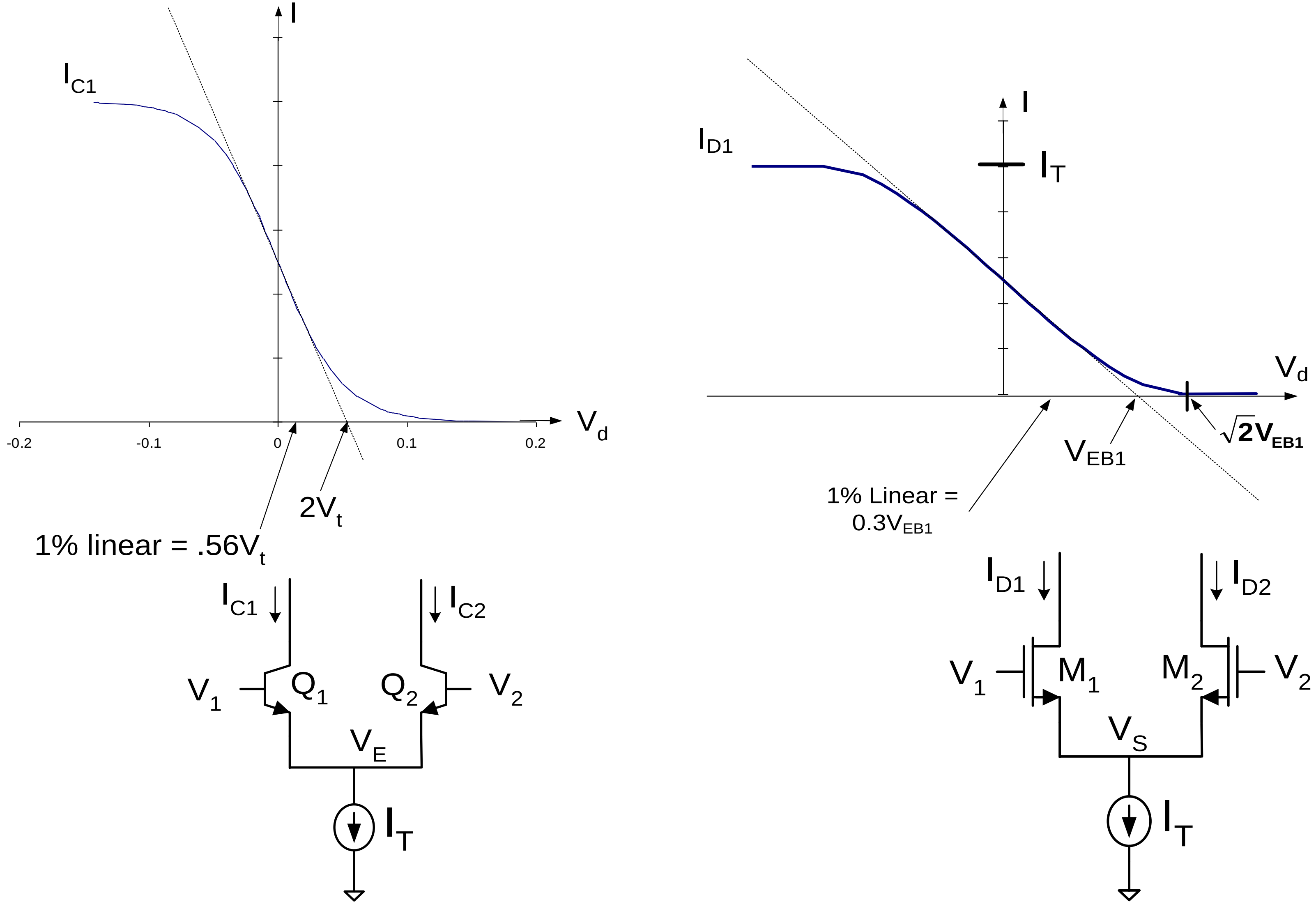
$$THD = -20\log\left(48\left(\frac{V_t}{V_m}\right)^2 - 3\right)$$

$$THD = -20\log\left(32\left(\frac{V_{EB1}}{V_m}\right)^2 - 3\right)$$

V_m/V_t	THD (dB)
2.5	-13.4049
1	-33.0643
0.5	-45.5292
0.25	-57.6732
0.1	-73.6194
0.05	-85.6647
0.025	-97.7069
0.01	-113.625

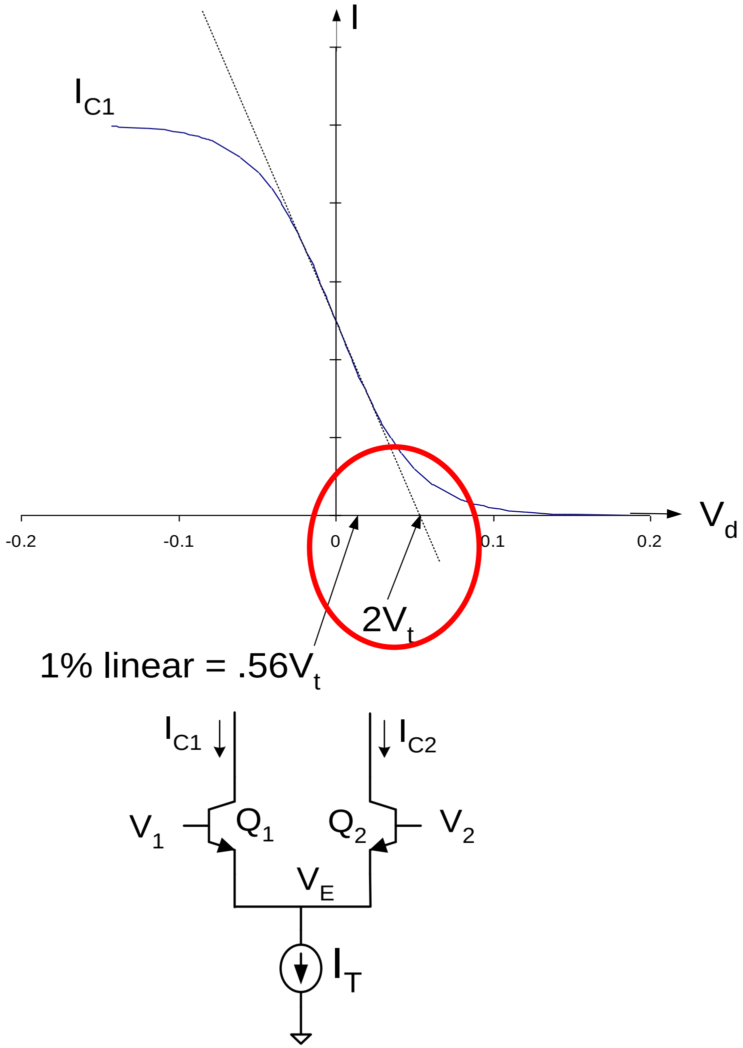
V_m/V_{EB1}	THD (dB)
2.5	-6.52672
1	-29.248
0.5	-41.9382
0.25	-54.1344
0.1	-70.0949
0.05	-82.1422
0.025	-94.1849
0.01	-110.103

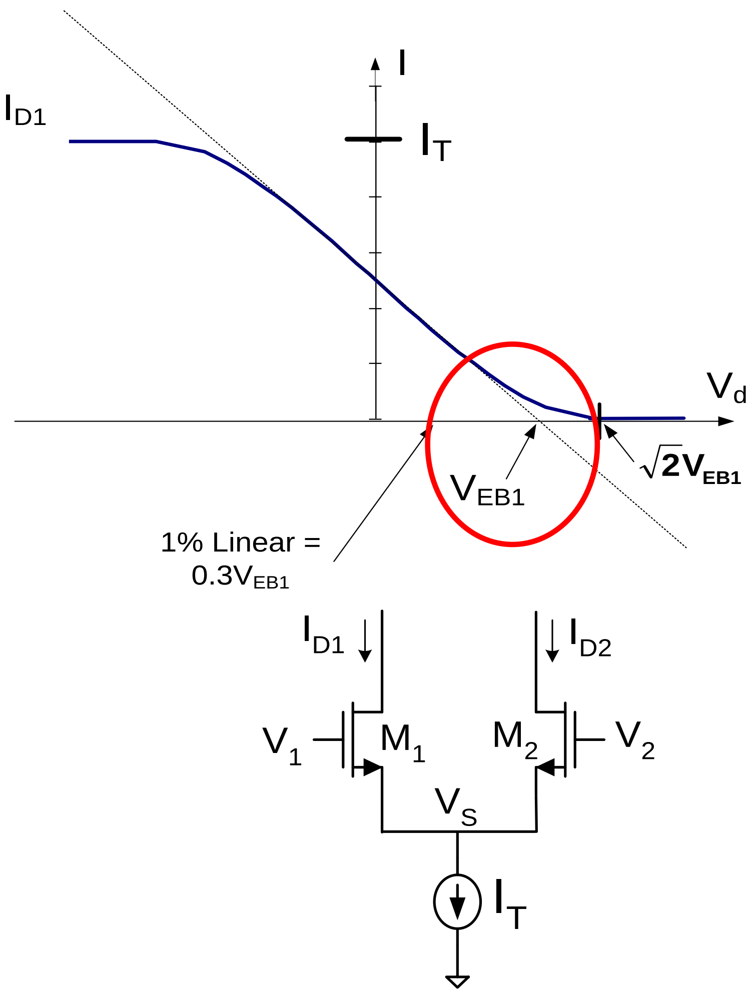
Linearity and Signal Swing Comparison of Bipolar/MOS Differential Pair



Have completed linearity analysis but must now look at the implications

Linearity and Signal Swing Comparison of Bipolar/MOS Differential Pair

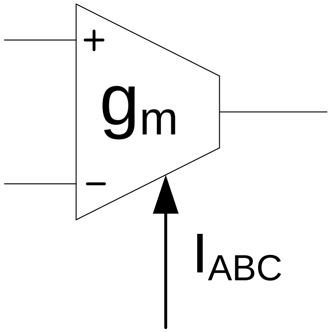




Signal swing determined by V_t

Signal swing determined by V_{EB}

Applications as a programmable OTA with I_{ABC}



The current-dependence of the g_m of the differential pair (single transistor) is often used to program the transconductance of an OTA with the tail bias current I_{ABC}

MOS

$$g_m = \sqrt{I_{ABC}}\sqrt{\mu C_{OX}\frac{W}{L}}$$

Two decade change in current for every decade change in g_m

$$g_m = uC_{OX}\frac{W}{L}V_{EB}$$

BJT

$$g_m = \frac{I_{ABC}}{2V_t}$$

One decade change in current for every decade change in g_m

What change in signal swing if programmed with I_{ABC}?

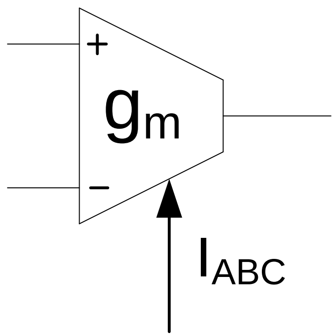
One decade decrease in signal swing for every decade decrease in g_m

No change in signal swing when g_m Is changed

Limited g_m adjustment possibility

Large g_m adjustment possible

Applications as a programmable OTA with I_{ABC}



MOS

$$g_m = \sqrt{I_{ABC}}\sqrt{\mu C_{OX}\frac{W}{L}}$$

One decade decrease in signal swing for every decade decrease in g_m

BJT

$$g_m = \frac{I_{ABC}}{2V_t}$$

No change in signal swing when g_m Is changed

Assume a MOS transconductor has an acceptable signal swing (as determined by linearity) with V_{EB}=1V (maybe p-p signal swing is V_{EB})

What would be the acceptable signal swing (with the same linearity) if g_m were tuned by one decade with I_{ABC}?

$$V_{EB1} = \sqrt{I_{DQ}}\sqrt{\frac{2L}{\mu C_{OX}W}}$$

$$V_{EB2} = \sqrt{\frac{I_{DQ}}{100}}\sqrt{\frac{2L}{\mu C_{OX}W}} = \frac{1}{10}\sqrt{I_{DQ}}\sqrt{\frac{2L}{\mu C_{OX}W}} = \frac{V_{EB1}}{10}$$

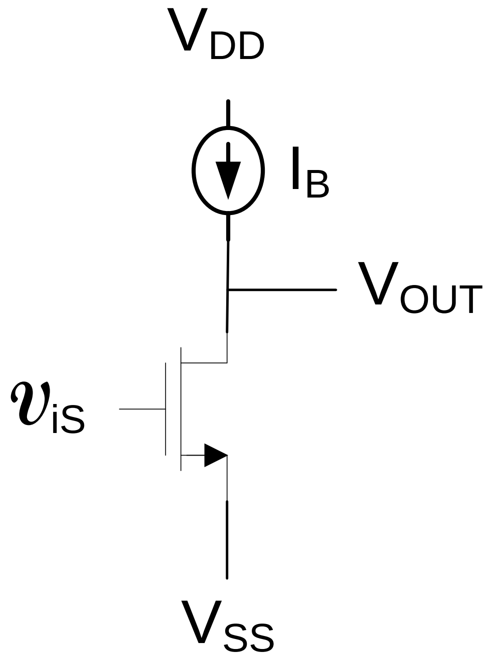
Signal swing would be reduced by a factor of 10

Signal Swing and Linearity Summary

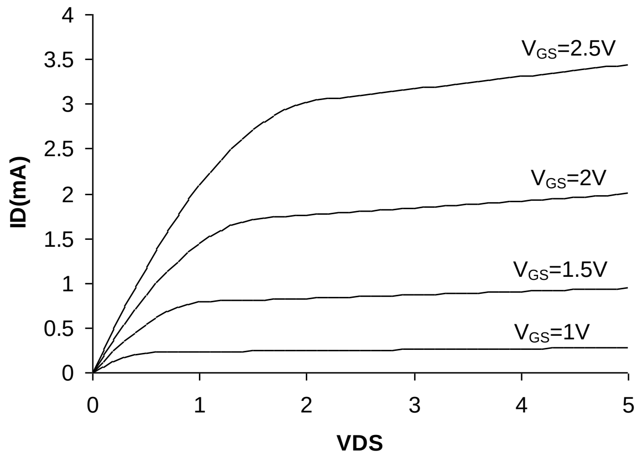
- Signal swing of MOSFET can be rather large if V_{EB} is large but this limits gain
- Signal swing of MOSFET degrades significantly if V_{EB} is changed for fixed W/L
- Bipolar swing is very small but independent of g_m
- Multiple-decade adjustment of bipolar g_m is practical
- Even though bipolar input swing is small, since gain is often very large, this small swing does usually not limit performance in feedback applications

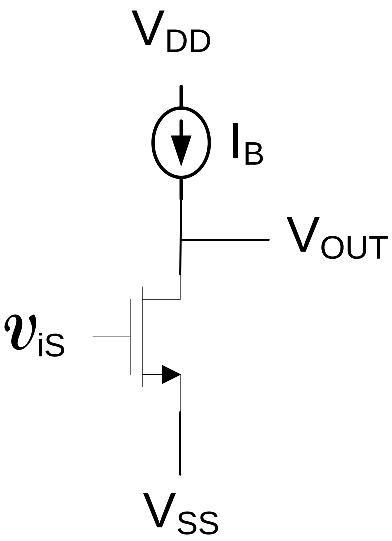
Linearity of Common-Source Amplifier

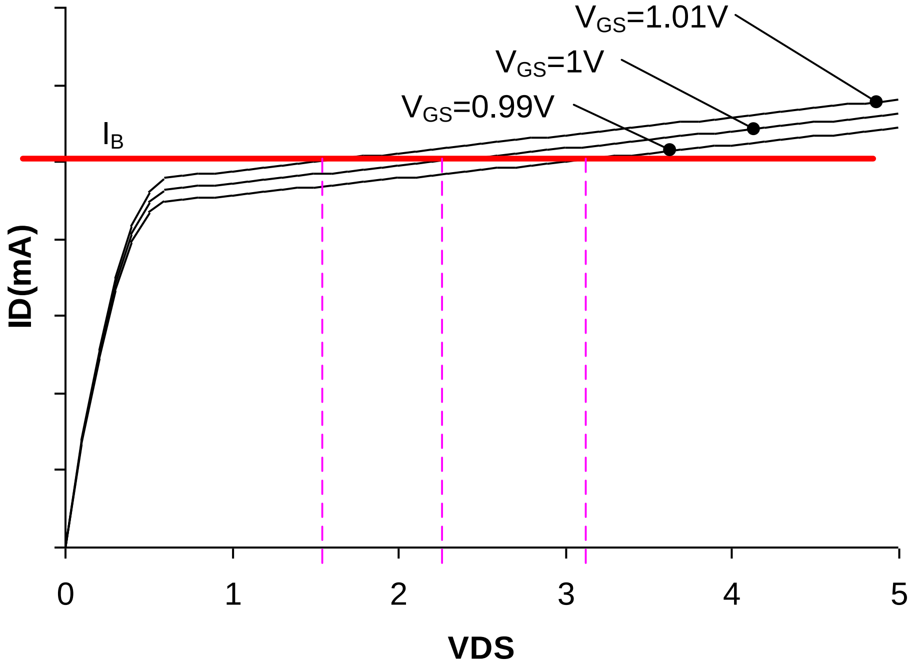
For convenience, will consider situation where current source biasing I_B is ideal



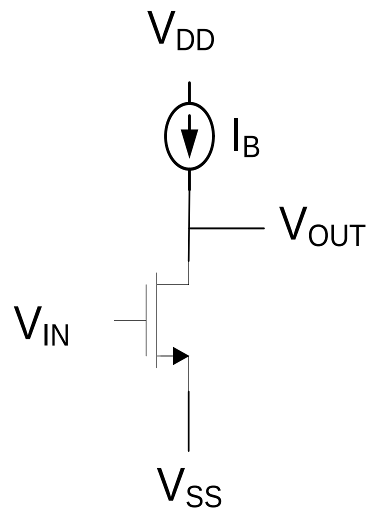
Linearity of Common-Source Amplifier







Linearity of Common-Source Amplifier



$$V_{IN} = V_{INQ} + v_{iS}$$

V_{INQ}: Quiescent Input
v_{iS}: Signal Input

$$V_{OUT} = V_{OQ} + v_{OS}$$

V_{OQ}: Quiescent Output
v_{OS}: Signal Output

$$I_B = \frac{\mu C_{OX} W}{2L}\left(V_{IN}-V_{SS}-V_{TH}\right)^2\left(1+\lambda\left[V_{OUT}-V_{SS}\right]\right)$$

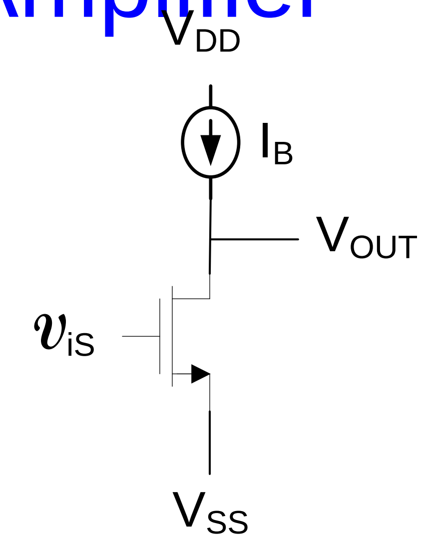
$$V_{EB} = V_{INQ} - V_{SS} - V_{TH}$$

strictly for notational convenience define $\beta = \frac{\mu C_{OX} W}{2L}$

$$I_B = \beta\left(v_{iS}-V_{EB}\right)^2\left(1+\lambda\left[v_{OS}+V_{OQ}-V_{SS}\right]\right)$$

$$v_{OS} = V_{SS}-V_{OQ}-\frac{\left(\dfrac{I_B}{\beta V_{EB}^2\left(1-\dfrac{v_{iS}}{V_{EB}}\right)^2}\right)-1}{\lambda}$$

Linearity of Common-Source Amplifier



$$v_{OS} = V_{SS} - V_{OQ} - \frac{\left(\dfrac{I_B}{\beta V_{EB}^2 \left(1 - \dfrac{v_{iS}}{V_{EB}}\right)^2}\right) - 1}{\lambda}$$

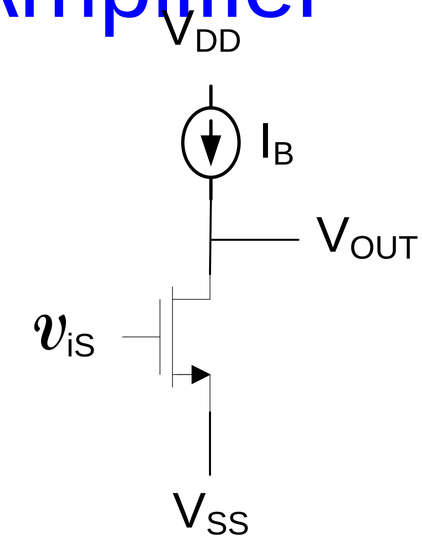
Recall for x small $\dfrac{1}{1+x} \simeq 1 - x$

$$v_{OS} \cong V_{SS} - V_{OQ} - \frac{\left(\dfrac{I_B \left(1 + \dfrac{v_{iS}}{V_{EB}}\right)^2}{\beta V_{EB}^2}\right) - 1}{\lambda}$$

$$v_{OS} \cong V_{SS} - V_{OQ} - \frac{I_B}{\lambda \beta V_{EB}^2}\left(1 + 2\frac{v_{iS}}{V_{EB}} + \left(\frac{v_{iS}}{V_{EB}}\right)^2\right) - \frac{1}{\lambda}$$

$$v_{OS} \cong \left[V_{SS} - V_{OQ} - \frac{1}{\lambda}\left(\frac{I_B}{\beta V_{EB}^2} + 1\right)\right] - \frac{I_B}{\lambda \beta V_{EB}^2}\left(2\frac{v_{iS}}{V_{EB}} + \left(\frac{v_{iS}}{V_{EB}}\right)^2\right)$$

Linearity of Common-Source Amplifier



$$v_{OS} \cong \left[V_{SS}\text{-}V_{OQ} - \frac{1}{\lambda}\left(\frac{I_B}{\beta V_{EB}^2}+1\right)\right] - \frac{I_B}{\lambda\beta V_{EB}^2}\left(2\frac{v_{iS}}{V_{EB}} + \left(\frac{v_{iS}}{V_{EB}}\right)^2\right)$$

but $$\left[V_{SS}\text{-}V_{OQ} - \frac{1}{\lambda}\left(\frac{I_B}{\beta V_{EB}^2}+1\right)\right] \cong 0$$

$$I_B \cong \beta(V_{EB})^2$$

Thus

$$v_{OS} \cong -\left(2\frac{v_{iS}}{\lambda V_{EB}} + \frac{1}{\lambda}\left(\frac{v_{iS}}{V_{EB}}\right)^2\right)$$

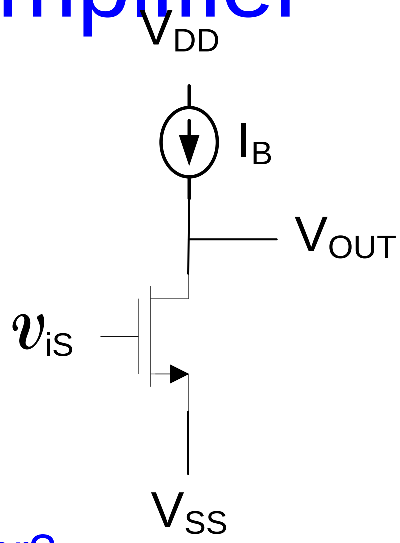
$$v_{OS} \cong -\frac{2}{\lambda V_{EB}}\left(v_{iS} + \frac{1}{2V_{EB}}v_{iS}^2\right)$$

Is this a linear or nonlinear relationship?

What are the dominant harmonics in the distortion of this amplifier?

Linearity of Common-Source Amplifier

$$v_{OS} \cong -\frac{2}{\lambda V_{EB}}\left(v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2\right)$$



What are the dominant harmonics in the distortion of this amplifier?

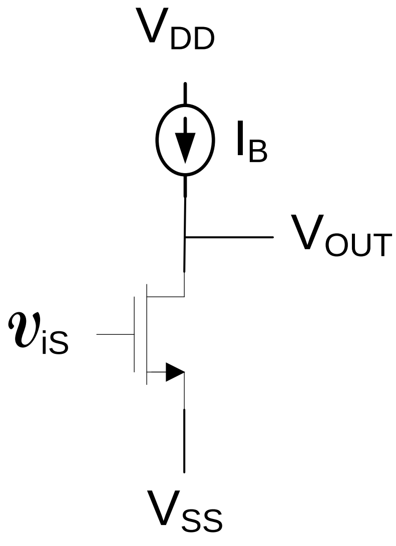
Consider input $V_m \sin(\omega t)$

Recall $$\sin^2 x = \frac{1-\cos 2x}{2}$$

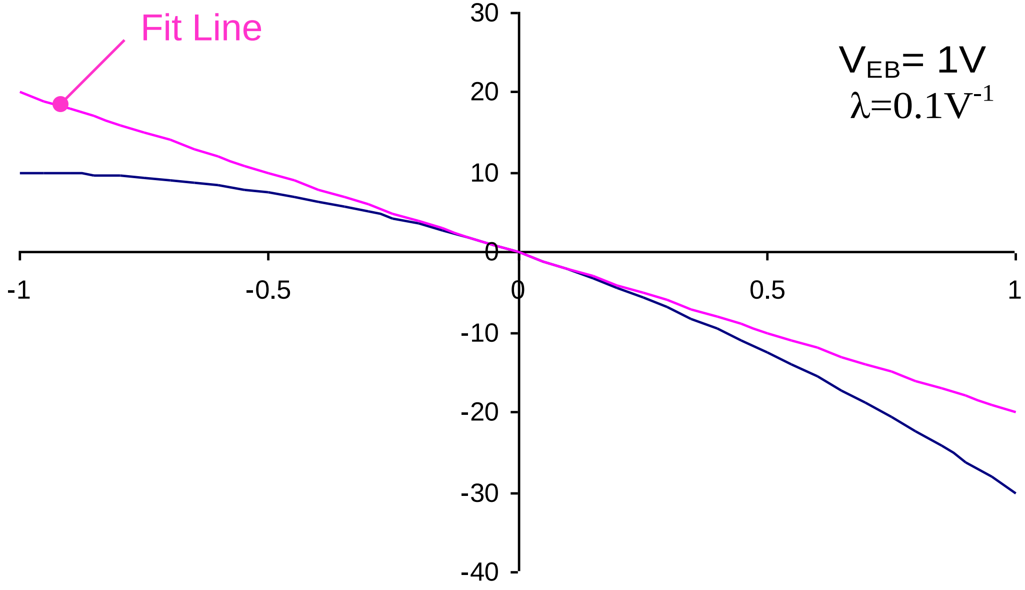
- Output will have components at ω and 2ω
- Dominant distortion is 2nd-order distortion
- This is in contrast to the differential pair that had dominantly 3rd order distortion
- Can readily obtain expression for THD

Linearity of Common-Source Amplifier

$$v_{OS} \cong -\frac{2}{\lambda V_{EB}}\left(v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2\right)$$



Is this a linear or nonlinear relationship?



when $v_{iS} = -V_{EB}$ (the minimum value of v_{iS} to maintain saturation operation)

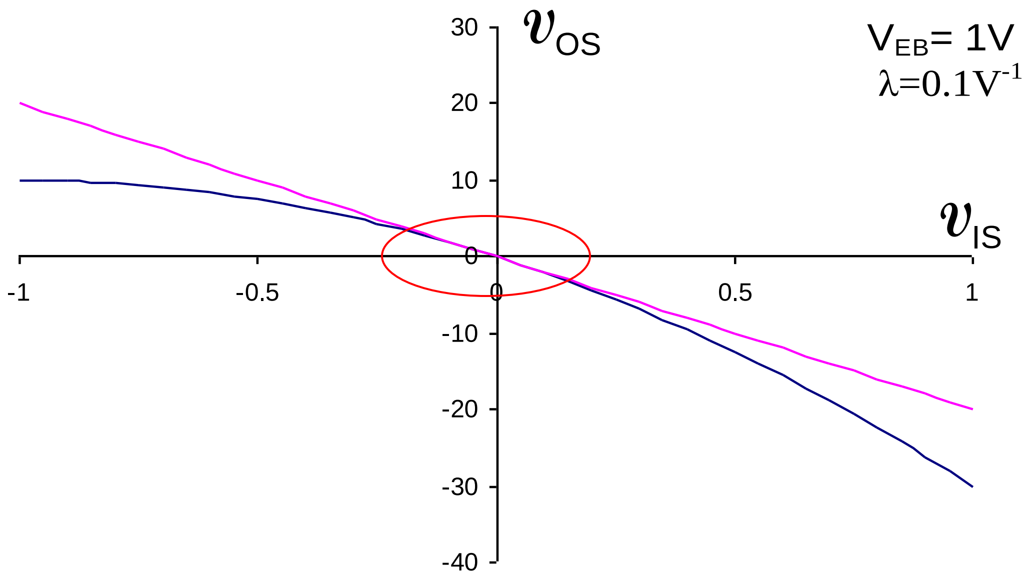
the error in V_{OS} will be $V_{EB}/2$ which is -50% !

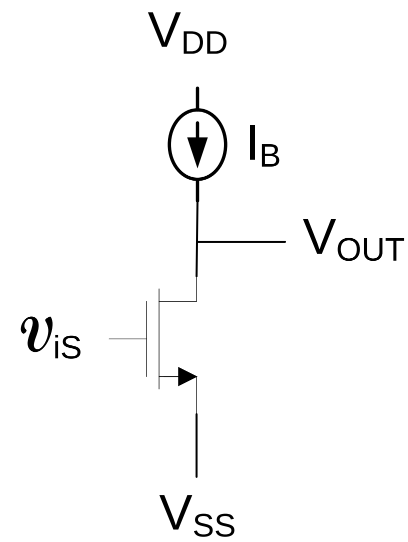
Is this a linear or nonlinear relationship?

Linearity of Common-Source Amplifier

$$v_{OS} \cong -\frac{2}{\lambda V_{EB}}\left(v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2\right)$$

Is this a linear or nonlinear relationship?





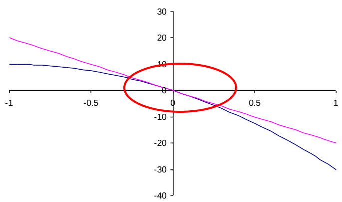
Note this is a reasonably high gain amplifier and could be larger for smaller V_{EB}

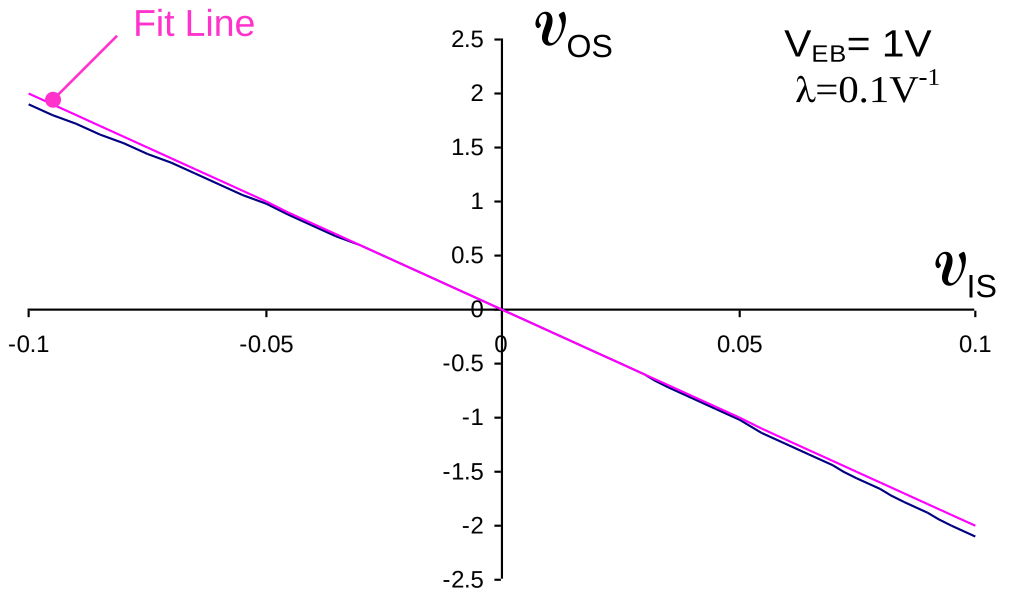
Over what output voltage range are we interested?

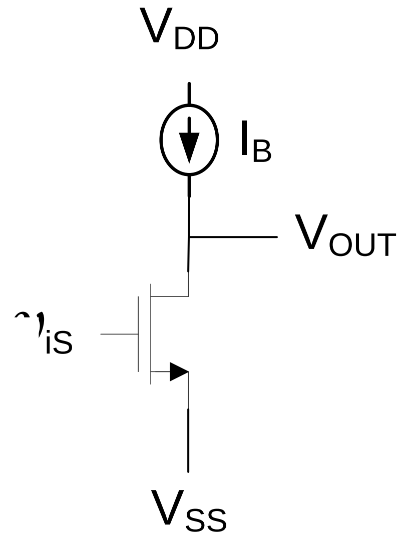
Linearity of Common-Source Amplifier

$$v_{OS} \cong -\frac{2}{\lambda V_{EB}}\left(v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2\right)$$

Is this a linear or nonlinear relationship?





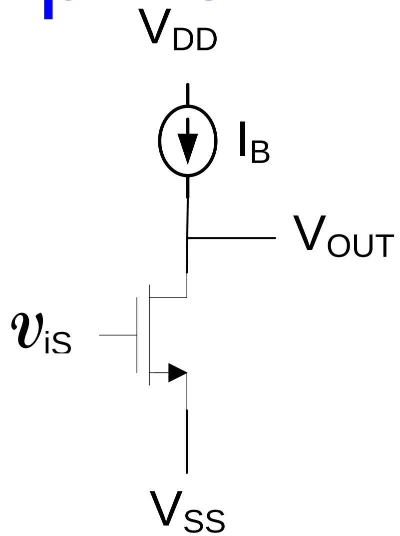


Linearity is reasonably good over practical input range

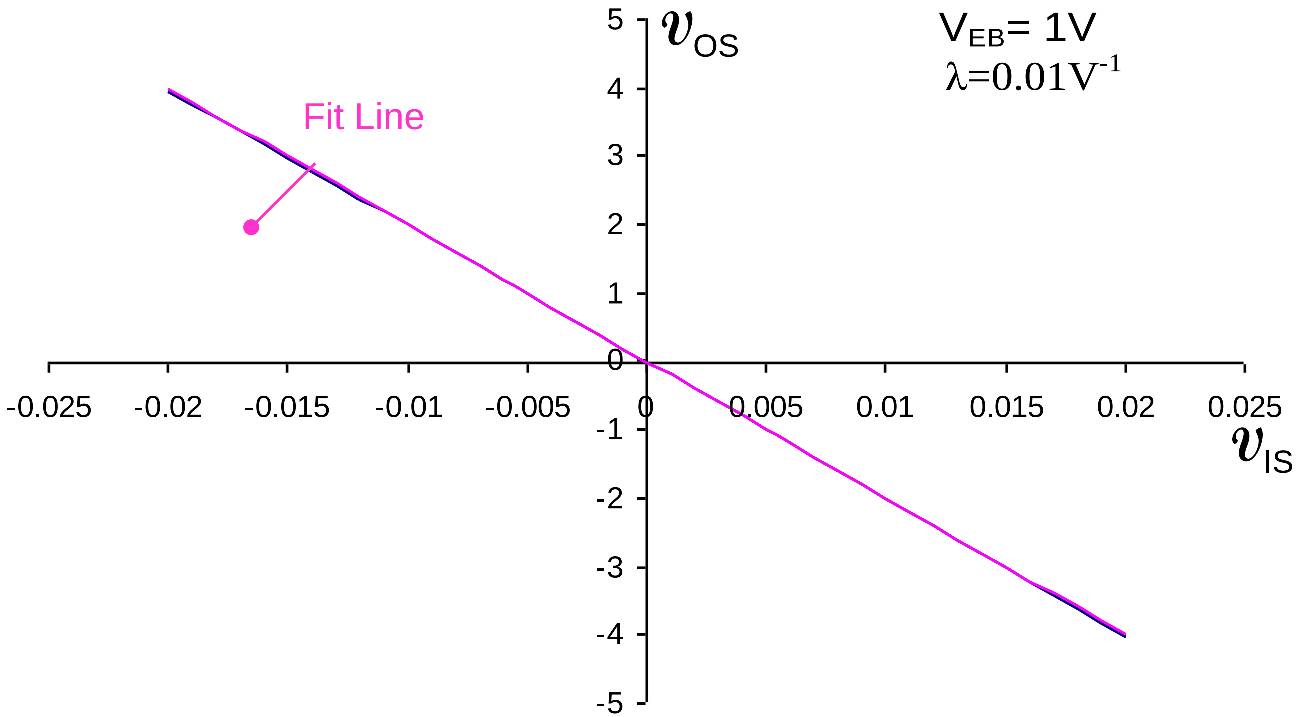
Practical input range is much less than V_{EB}

Linearity of Common-Source Amplifier

$$v_{OS} \cong -\frac{2}{\lambda V_{EB}}\left(v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2\right)$$



Is this a linear or nonlinear relationship?



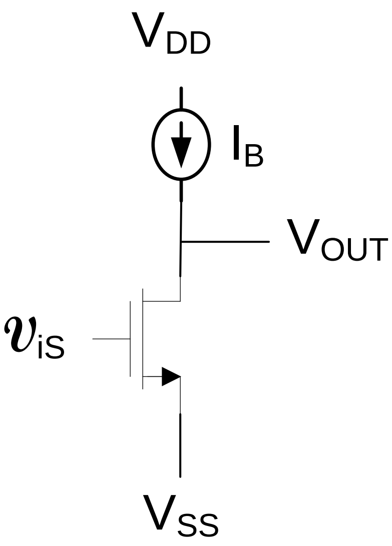
$\lambda=0.01V^{-1}$ more realistic in many processes or for longer L

Can't "see" nonlinearity in this plot

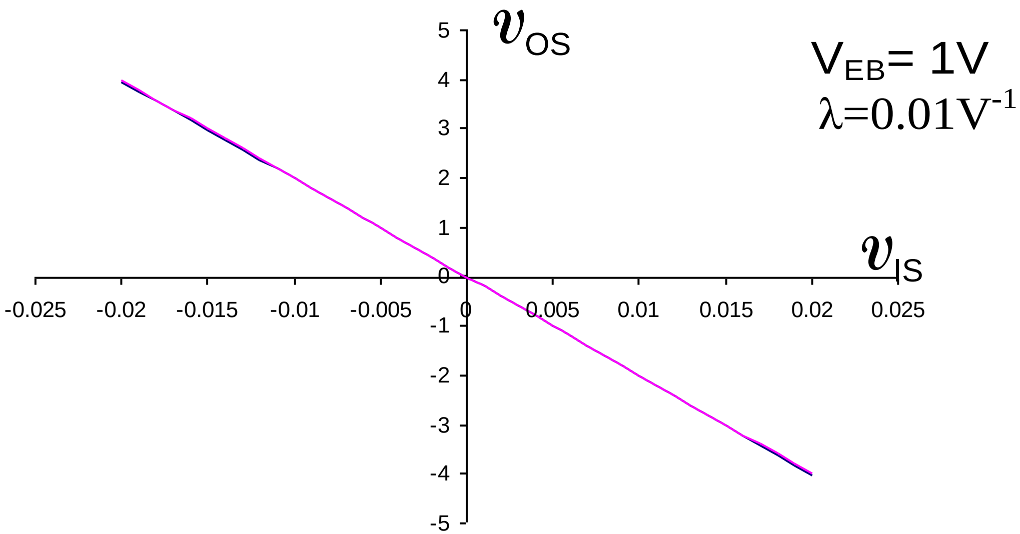
Appears to be dependent upon dc gain of amplifier ??

Linearity of Common-Source Amplifier

$$v_{OS} \cong -\frac{2}{\lambda V_{EB}}\left(v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2\right)$$



Is this a linear or nonlinear relationship?



Will look at difference between output and ideal output as defined by fit lie

$$v_{FIT} \cong -\frac{2}{\lambda V_{EB}} v_{iS}$$

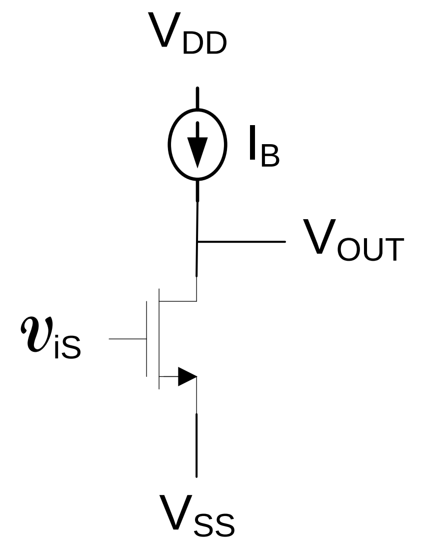
$$\varepsilon = v_{FIT} - v_{OS}$$

$$\varepsilon \cong \frac{1}{\lambda V_{EB}^2} v_{iS}^2$$

Appears to be highly dependent upon dc gain of amplifier ??

Linearity of Common-Source Amplifier

$$v_{OS} \cong -\frac{2}{\lambda V_{EB}}\left(v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2\right)$$

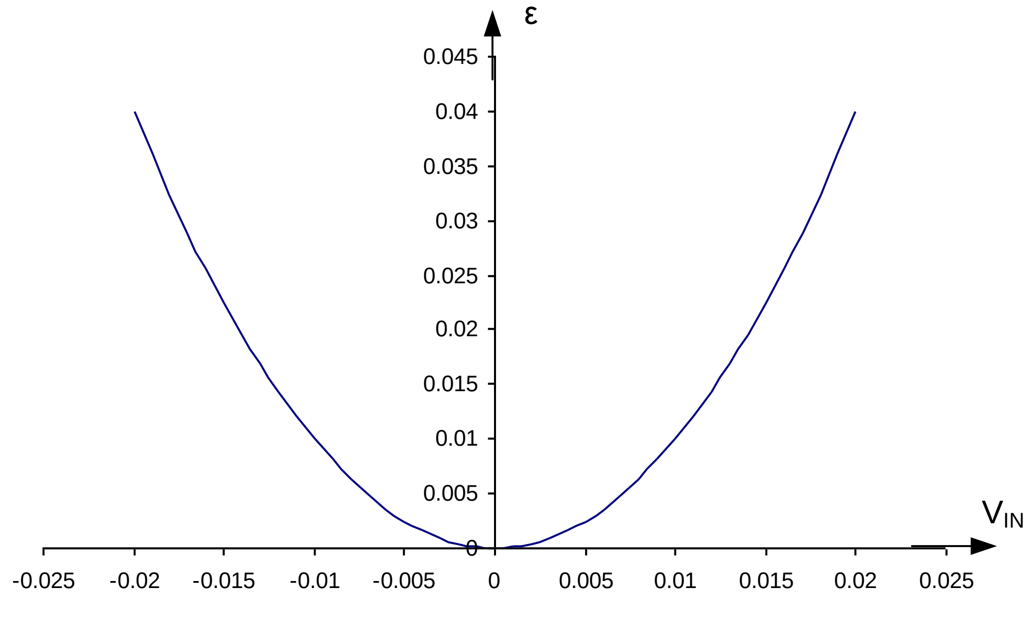


Is this a linear or nonlinear relationship?

$$\varepsilon \cong \frac{1}{\lambda V_{EB}^2} v_{iS}^2$$

V_{EB}= 1V

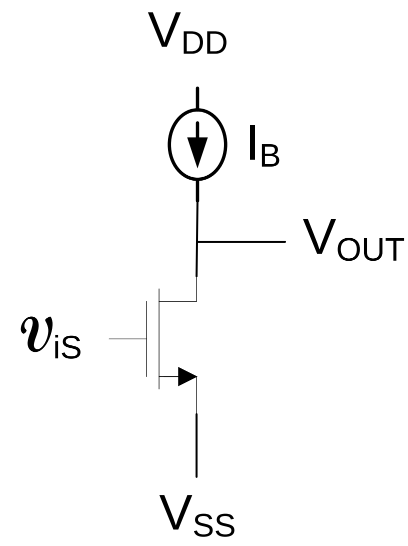
λ=0.01V^{-1}



Appears to be highly dependent upon dc gain of amplifier ??

Linearity of Common-Source Amplifier

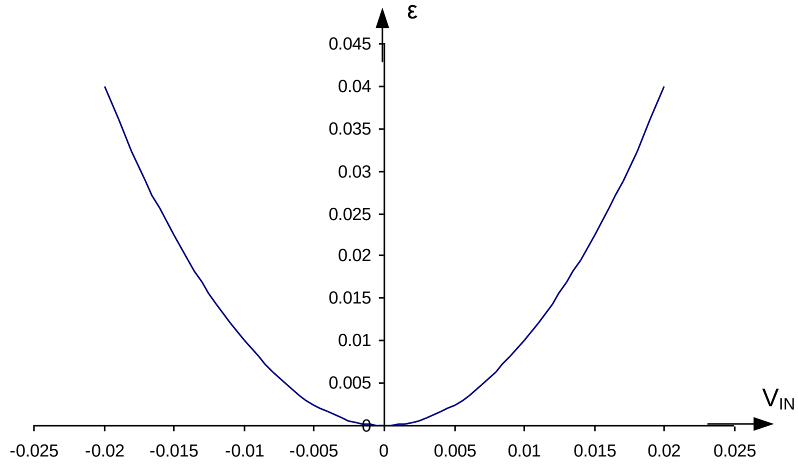
$$v_{OS} \cong -\frac{2}{\lambda V_{EB}}\left(v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2\right)$$



Is this a linear or nonlinear relationship?

$V_{EB} = 1\text{V}$

$\lambda = 0.01\text{V}^{-1}$



$$\varepsilon_{PCT} \cong \frac{\varepsilon}{v_{FIT}} 100\% = \left[\frac{\frac{1}{\lambda V_{EB}^2} v_{iS}^2}{\frac{2 v_{iS}}{\lambda V_{EB}}}\right] 100\% = \left(\frac{100\%}{2V_{EB}}\right) v_{iS}$$

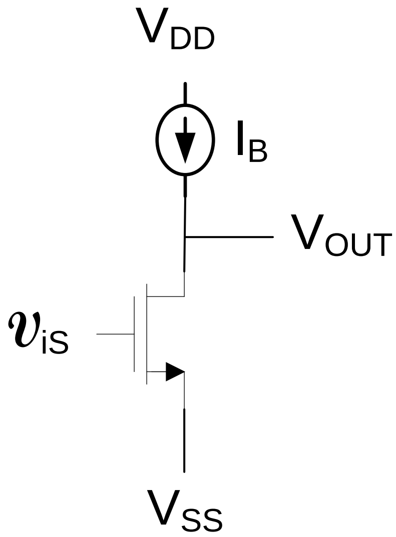
$$\varepsilon_{PCT} \cong \left(-\frac{\lambda \bullet 100\%}{4}\right) v_{OS}$$

Appears to be highly dependent upon dc gain of amplifier ??

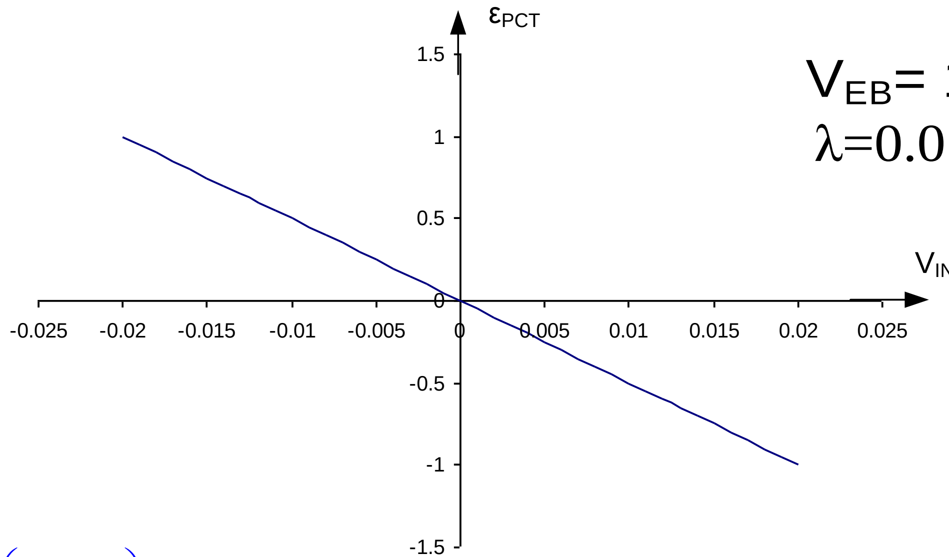
Relative error in output independent of gain of amplifier !

Linearity of Common-Source Amplifier

$$v_{OS} \cong -\frac{2}{\lambda V_{EB}}\left(v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2\right)$$



Is this a linear or nonlinear relationship?



$V_{EB} = 1V$

$\lambda = 0.01V^{-1}$

$$\varepsilon_{PCT} \cong \left(\frac{100\%}{2V_{EB}}\right) v_{iS}$$

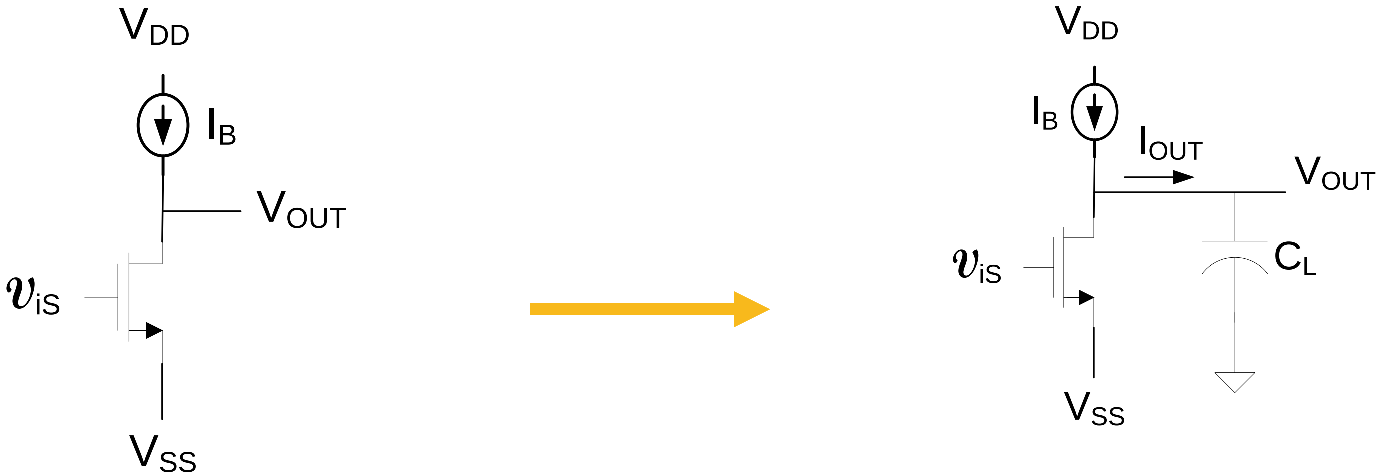
or, in terms of v_{OS},

$$\varepsilon_{PCT} \cong \left(-\frac{\lambda \bullet 100\%}{4}\right) v_{OS}$$

1% deviation for this example occurs at $\left|v_{OS}\right| \cong 0.01\frac{4}{\lambda} \cong 4V$

In spite of square-law nonlinearity in MOSFET, linearity of CS amplifier is quite good provided MOSFET remains in saturation region !!

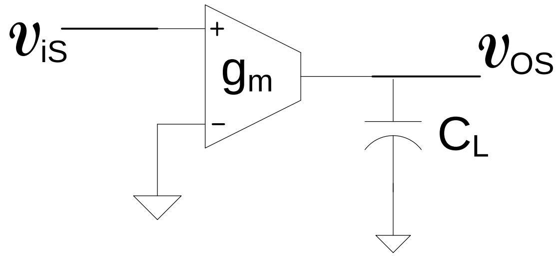
Linearity of Common-Source Amplifier



High-Gain Amplifier

Transconductance Amplifier

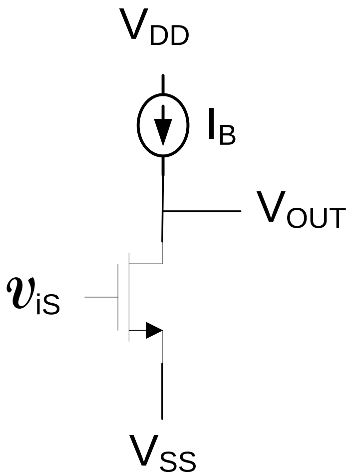
The transconductance amplifier driving a load C_L is performing as an integrator



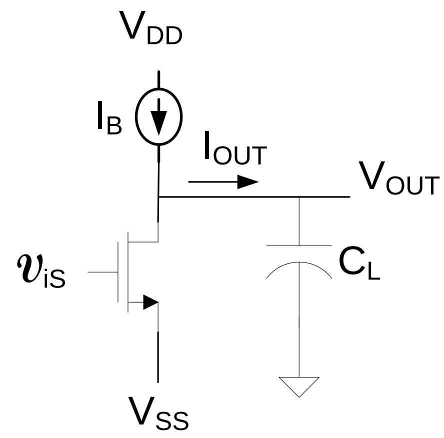
Integrators often used in filters where at frequencies of most interest $|v_{OS}|$ is comparable to $|v_{iS}|$

Is this common-source amplifier linear or nonlinear?

Linearity of Common-Source Amplifier



High-Gain Amplifier



Transconductance Amplifier

$$I_{OUT} = I_B - I_D$$

$$I_{OUT} = I_B - \beta(v_{iS} + V_{EB})^2\left(1+\lambda\left[V_{OS}+V_{OQ}-V_{SS}\right]\right)$$

$$I_{OUT} = \left[I_B - \beta(V_{EB})^2\left(1+\lambda\left[V_{OQ}-V_{SS}\right]\right)\right] - \beta\left(v_{iS}^2 + 2v_{iS}V_{EB}\right)\left(1+\lambda\left[V_{OS}+V_{OQ}-V_{SS}\right]\right)$$

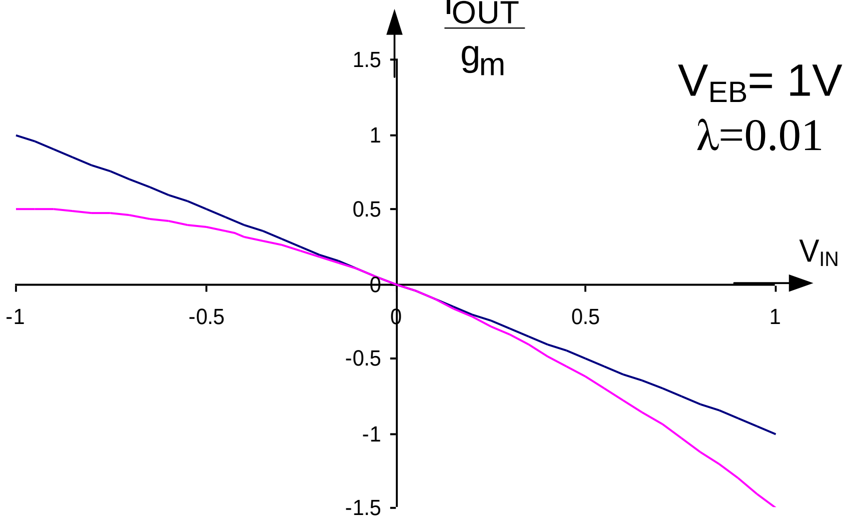
$$I_{OUT} \cong -\beta\left(v_{iS}^2 + 2v_{iS}V_{EB}\right)$$

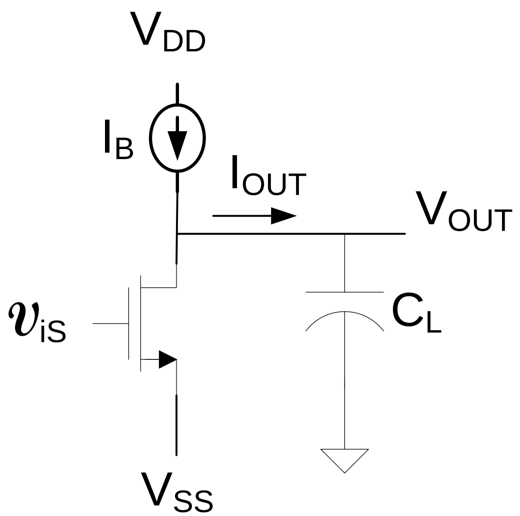
$$I_{OUT} \cong -\frac{2I_B}{V_{EB}}\left(v_{iS} + \frac{1}{2V_{EB}}v_{iS}^2\right)$$

Is this a linear or nonlinear relationship?

Linearity of Common-Source Amplifier

As an OTA





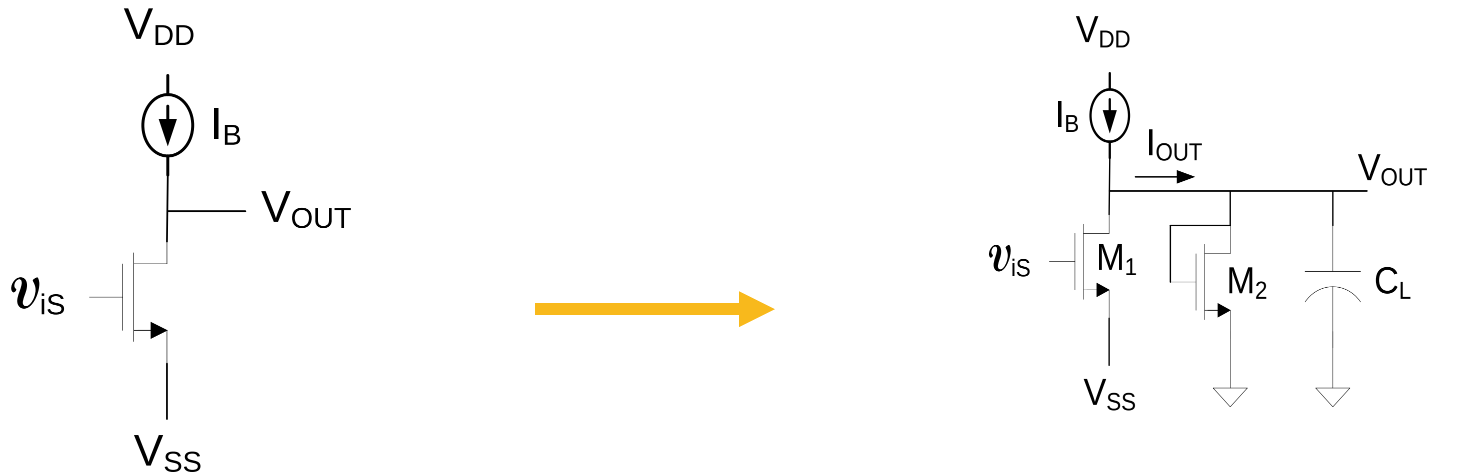
$$g_m = \frac{2I_B}{V_{EB}}$$

$$I_{OUT} \cong -\frac{2I_B}{V_{EB}}\left(v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2\right)$$

Is this a linear or nonlinear relationship?

At v_{IS}=-V_{EB}, the error in I_{OUT} will be -50% !

Linearity of Common-Source Amplifier



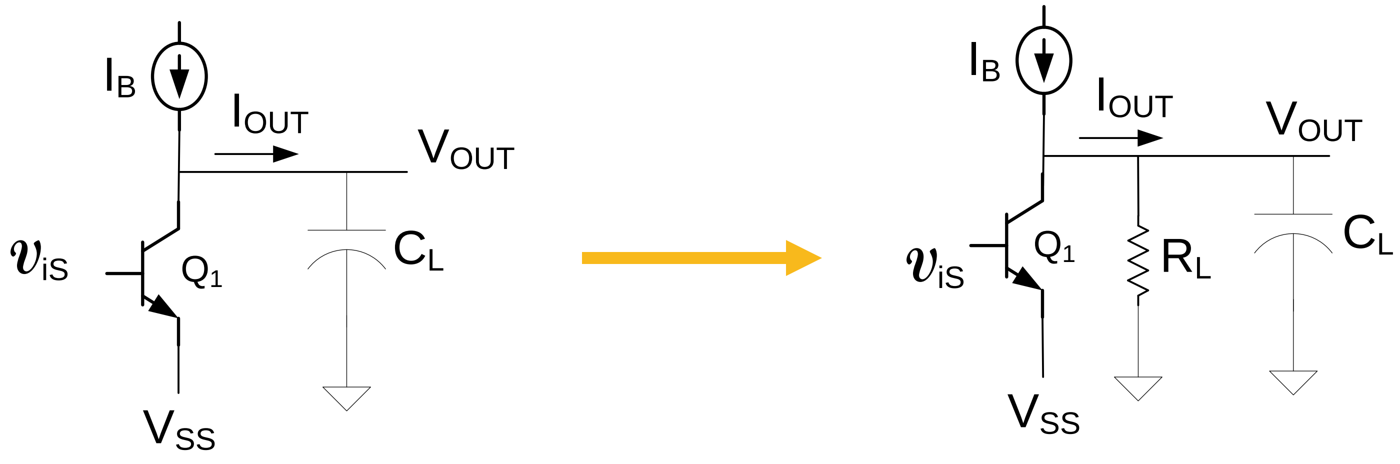
High-Gain Amplifier

Transconductance Amplifier

Is this common-source amplifier linear?

- Reasonably linear if used in high-gain applications and V_{EB} is large (e.g. if $A_V=g_m/g_o=2/((\lambda V_{EB})=100$ and Vo=1V, Vin=10mV)
- Highly nonlinear when used in low-gain applications though linearity dependent upon g_m

Linearity of Common-Emitter Amplifier



High-Gain Amplifier

Transconductance Amplifier

Is this common-emitter amplifier linear?

- Very linear if used in high-gain applications
  (e.g. if $A_V=g_m/g_0=V_{AF}/V_t=4000$ and $V_o=1V$, $V_{in}=250uV$)

- Highly nonlinear when used in low-gain applications but not dependent upon g_m

- Bipolar OTAs (e.g. current mirror op amp) can operate over multiple decades of gain with low-level signals but no degradation with gain

Stay Safe and Stay Healthy !

End of Lecture 21